STABILITY OF LARGE SCALE POWER SYSTEMS

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Dedicated to
my beloved parents and wife.

CERTIFICATE

Certified that this work, 'Stability of Large Scale Power Systems' by Chivukula Lakshminarayana has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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LIST OF PRINCIPAL SYMBOLS

M_i : Angular Momentum of the ith machine

H_i : Inertia constant of the ith machine

D_i : Damping coefficient of the ith machine

E_i : Magnitude of the voltage behind the transient reactance of the ith machine

P_{mi} : Mechanical power input to the machine 'i'

P_{ei} : Electrical power output of machine 'i'

δ_i : Rotor angle of the machine 'i' with respect to a rotating frame of reference

δ_i^o : Post-fault steady state rotor angle δ of machine i

δ_i^s : Pre-fault steady state rotor angle δ of machine i

$\bar{Y}_{ij}$: The ijth term of the admittance matrix Y and of the form

$$\bar{Y}_{ij} = Y_{ij} \angle\theta_{ij} = G_{ij} + jB_{ij}$$

λ_i : The ratio D_i/M_i

$\underline{X}$: State vector of the appropriate order

ω_i : angular velocity of machine 'i'

x_i : transient reactance of machine 'i'

A dot over a symbol denotes differentiation with respect to time.

A bar below a symbol signifies a vector.

A superscript T denotes 'Transpose'.

Double subscript notation denotes the order of the corresponding matrix.

SYNOPSIS

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STABILITY OF LARGE SCALE POWER SYSTEMS

Transient stability of a power system is an attribute of the system which denotes the conditions in which the various synchronous machines of the system remain in synchronism with each other when sudden disturbances occur on the system. Stability studies are necessary for planning new facilities for future load growth and also for a reliable operation of the system. For such studies it is now acknowledged that Lyapunov method yields satisfactory results as compared with the conventional method using repeated integration of the system equations for different assumed clearing times. Ever since the original works of El-Abiad and Nagappan, Gless and Aylett in this area there has been considerable interest in the use of Lyapunov functions for assessing power system stability. However, after 1972, the interest somewhat declined due to some of the practical difficulties encountered in the successful application of this method to realistic power systems.

Some of these are:

(i) Determination of the stability boundary for the post-fault system. This involved the computation of $2^{n-1}-1$ unstable equilibrium points. This was a formidable task even for off line studies,

(ii) The larger size of the system necessitated the use of simpler models for the synchronous machines,

and (iii) Increased interconnection of the system required the construction of the dynamic equivalents and their incorporation in Lyapunov methods.

In this thesis solution to some of these problems is presented. As regards the determination of the stability boundary the recent work of Prabhakara and El-Abiad marks a big step forward. However, it has certain drawbacks in terms of choice of reference machine and an adequate theoretical basis. In this thesis the problem of determining the stability regions as formulated in the context of a multinonlinear Lure'-Popov system where the nonlinearities satisfy the sector condition only in a region around the origin of the state space. Utilizing this sector violation information of the nonlinearities, a generalization of an earlier result due to Walker and Mc'Clamroch and Weissenberger is obtained. This result

is further suitably modified to suit its application to the power system model. The method of Prabhakara and El-Abiad is shown to be a special case of the general result obtained.

The problem of handling large scale power systems is analyzed by suggesting a method of decomposition and the use of a new theoretical concept of vector Lyapunov functions. The large scale power system is decomposed into low order models called the 'subsystems' and the stability properties of these subsystems are analyzed separately by constructing scalar Lyapunov functions for each of these subsystems. On the higher hierarchial level these scalar functions are used to define a vector Lyapunov function for the composite system. Using this vector function together with the nature of interactions among the subsystems, the stability of the overall system is inferred. A 3 machine example illustrates the application of the proposed method.

A third contribution of the thesis is in the area of using energy or energy type Lyapunov functions to identify coherency and also subsequently develop dynamic equivalents for large power systems. Stability analysis by both conventional and Lyapunov based methods is carried out for the reduced and the unreduced systems, and the results are compared.

The following is a chapterwise summary of the work reported in this thesis:

The first chapter introduces the problem of stability analysis of large power systems using the second method of Lyapunov. The state of the art is briefly reviewed and then the scope and objective of the thesis are outlined.

The second chapter is devoted to the development of state space models of power systems. Although a proper rationale involving the control theoretic concepts of controllability and observability has been given for a proper choice of the state variables, the reasons for such uncontrollability are not evident. In this chapter it is shown that this state uncontrollability is partially due to the state overdescription in the model and hence the corresponding state vector is not of the proper order. A legitimate order of the state vector is obtained and is shown to be the same as that obtained by the control theoretic approach. Thus some additional light is thrown on this already settled controversy on the minimal order of the system.

The third chapter introduces a new concept of vector Lyapunov functions for the stability of a large scale power systems. The theory of vector Lyapunov

function is developed to suit the power system problem. A decomposition of the power system into lower order subsystems is obtained. Each of these subsystems is shown to be identical in form and in order. Lyapunov functions for each of these subsystems are constructed using the Moore-Anderson's theorem. These functions have a positive definite quadratic form with a negative definite derivative. A vector Lyapunov function is defined for the composite system. Conditions for the stability of the overall system are then derived. A scalar Lyapunov function is also defined for the composite system and is proved to possess a negative definite derivative. This scalar function is subsequently used for determining the stability domain. Illustrative example of a three machine case is given.

The fourth chapter deals with the problem of obtaining stability region for a multimachine power system without computing the unstable equilibrium points. The problem is posed first as a multinonlinear problem of the Lure'-Popov type in which the nonlinearities violate their respective Popov sectors. Using this sector violation information, an expression for the stability boundary is obtained constituting a generalization of the results of Walker and Mc'Clamroch obtained for a system with a single nonlinearity. The technique

is then extended to a multimachine power system for estimating the stability domain. Two algorithms are proposed for practical implementations. The methods are illustrated with reference to a 5-machine example.

The fifth chapter addresses itself to the problem of stability analysis using reduced order models of the power system. A criterion using energy functions for identifying coherent group and for the development of the dynamic equivalent is proposed. Test results on a 44 bus system with 15 generators are presented.

A general review of the results of this research, identification of some unsolved problems and suggestions for future work form the theme of the concluding sixth chapter.

CHAPTER I

INTRODUCTION

1.1 THE STABILITY PROBLEM:

The term 'Stability' when used with reference to a power system is that property of the system or a part of the system which enables it to develop restoring forces between its elements equal to or greater than the disturbing forces so as to restore a state of equilibrium between its elements [1]. (American Standards Definitions of Electrical terms, ASA-C42-1941). Although stability is a single phenomenon, there are two important stability concepts, namely, the local stability and the transient stability. Whereas the former, usually referred to as 'Steady state stability', deals with the stability of the equilibrium state of the system for small deviations of the system variables such as the load disturbances or prime mover inputs, the latter is concerned with the stability and transmittal of power when the system is subjected to a severe 'aperiodic disturbance'. By an aperiodic disturbance, it is meant that it does not come with regularity but occurs after an interval during which the system reaches an equilibrium state between disturbances [1]. These disturbances may be due to sudden loss of load, loss of excitation, switching operation or sudden short circuits. Of these the short circuit is the most severe and, unless it is cleared

quickly, results in a loss of synchronism of the system. It is therefore evident that there exists a maximum time beyond which the fault cannot be allowed to persist on the system. This is called the critical clearing time. An assessment of this critical clearing time is extremely important in planning studies, relay coordination studies etc.

The problem of transient stability is therefore formulated in the following manner: Given a system, does there exist an equilibrium state of the system after the disturbance is cleared ? If yes, what is the maximum time that the disturbance may be allowed to remain without the system losing synchronism ? An analysis of the problem essentially involves the following two main steps:

Step 1: The study of the evolution of the system from the occurrence of the disturbance to the time of clearing.

Step 2: The study of the evolution of the system beyond the time of clearing.

The intervals of time corresponding to these two steps are called the 'faulted state' and the 'post-fault state' respectively. Each of these steps involves the integration of a large number of nonlinear differential equations.

The topic of transient stability investigation dates back to the early 1920's when the point-by-point method [2]

was used for the analysis of simple 2 or 3 machine systems. Subsequently, the network analyzer was used to simulate larger sized systems for carrying out these investigations. With the increase in size and complexity of the power system, the digital computer has attained undisputed supremacy over the network analyzer for such and other studies.

The classical method of stability investigation consists in assuming a clearing time 't_e' arbitrarily and solving the differential equations of the system in its faulted state upto t_e and the post fault state after t_e to obtain the swing curves. The initial conditions are obtained from a prefault load flow data. These swing curves which are a plot of the rotor angles versus time are examined for the stability of the post-fault system. This is usually done by observing whether the rotor angle differences with respect to any one machine taken as a reference tend to remain constant during the first swing of the rotor angles. In such a case we conclude that the system is stable. If they tend to diverge, then it is considered to be unstable. Generally it is assumed that if the system is stable during the first swing, it is also stable thereafter. This process is repeated for another clearing time. After a certain number of such repeat procedures, the clearing time at which the transition from the stable to the unstable state of the system takes place is established. This gives the

critical clearing time. Standard numerical integration and associated programs are available to carry out such investigations. Nevertheless, this procedure of direct determination of the transients is not an efficient way from the computational point of view when one looks for a quick information about the stability with respect to various possible perturbations. It is because of this reason, specific care has been devoted in the last few years to examine alternatives such as direct analytical methods based on Lyapunov theory.

Stability investigations by direct methods in contrast to the classical methods are carried out by making partial or no use of the system differential equations. The equal area criterion [2] is a graphical procedure for single and two machine systems and does not make use of the differential equations as such. Analytical methods such as the phase plane technique [3] were employed for single and two machine systems. The energy integral criterion [4] and the direct method of Lyapunov [5-35] for multimachine systems make use of the system differential equations partially. Of these, the Lyapunov's direct method has been a topic of active research in recent years. The technique essentially consists in establishing a region of stability around the post-fault equilibrium state via the construction of a Lyapunov function. The system is considered stable if

the solution trajectories of the faulted and the post-fault states lie entirely in this region [6].

1.2 LYAPUNOV'S DIRECT METHOD APPLIED TO POWER SYSTEMS:

The application of Lyapunov's direct method to the transient stability studies of multimachine power systems consists in the replacement of the step 2 detailed above by a stability criterion in the form of a Lyapunov function around the post-fault equilibrium point. This construction of a Lyapunov function, in fact, is a key step in the application of the Lyapunov theory to the power system stability problem. Such a criterion was first suggested by Aylett [4] in 1958 based on energetical considerations but the theory was not well developed then for practical implementation. The pioneering work of El-Abiad and Nagappan [6] and Gless Gless [5] triggered a series of investigations in this area. An excellent survey of the literature in connection with the construction of Lyapunov function is contained in references [22] and [11]. The latter also contains a critical review of these functions. Two different lines of approach have been adopted by researchers for the construction of the Lyapunov functions. They are (i) the Kalman's construction procedure used by Pai et al [9] and Pai [7] and (ii) the Anderson's technique applied by Willems [14,15] and Pai and Murthy [10]. In reference [10], a Lyapunov function was obtained with a quadratic part $\underline{X}^T P \underline{X}$ which

has a positive definite symmetric P. Such a function has been found useful in the application of vector Lyapunov function theory [52] which constitutes a part of this thesis. Recent works of Mansour [19] and Willems [21] generalize the results so far obtained in the literature via generalized Lyapunov functions.

The procedure of applying Lyapunov's second method to the power system stability problem consists in obtaining a suitable mathematical model for the system of the type

$$\dot{\underline{X}} = \underline{F}(\underline{X}) \tag{1.1}$$

for both the faulted and the post-fault states. Let these be represented by

$$\dot{\underline{X}} = \underline{F}_1(\underline{X}) \tag{1.2}$$

and $$\dot{\underline{X}} = \underline{F}_2(\underline{X}) \tag{1.3}$$

for the respective states. A Lyapunov function $V(\underline{X})$ is them constructed for the system (1.3). Then a stability region around the origin of the post-fault equilibrium state $\underline{X} = \underline{0}$ is defined by an inequality

$$V(\underline{X}) < \varepsilon \tag{1.4}$$

where ε is a constant to be computed. This constant is usually obtained by evaluating the function $V(\underline{X})$ at the unstable equilibrium point closest to the post-fault stable equilibrium point. Now the calculation of the critical clearing time is simple and is done by integrating

the differential equations (1.2) and evaluating $V(\underline{X})$ at every time step of the integration. The time t_c at which

$$V(\underline{X}) = \varepsilon \tag{1.5}$$

gives an estimate of the critical clearing time. Thus an explicit integration of the differential equations (1.2) is performed only once unlike the classical techniques. This results in a significant reduction in the computation time particularly in the analysis of large scale power systems for various contingencies. In this case one can consider the Lyapunov method as complementing the classical methods. A preliminary screening by this method can reduce the number of studies to be carried out in detail. Another advantage of the method lies in investigating the effects of parameter variations such as damping [7, 14], power inputs [23] etc., thoroughly and quickly. Stability indices can also be defined by this method without making actual stability calculations. It is now gradually being recognized that the method is able to give satisfactory results in agreement with those obtained by direct simulation.

1.3 THE LARGE SCALE POWER SYSTEM STABILITY PROBLEM:

The technique of applying Lyapunov methods has been described in the previous section. While considerable efforts have been directed towards the improvement of the Lyapunov functions [10, 19, 21] on the one hand, the

application of the theory to realistic systems on the other hand has been very slow inspite of the advantages mentioned earlier. This is due to the fact that there are certain drawbacks, apart from the general conservativeness of the method, that still need further attention. Firstly, the application of the theory to large power systems poses heavy computational problems in the determination of stability regions [35]. It has been established that for an n-machine system, the number of unstable equilibrium points is $2^{n-1}-1$, and consequently very large for a large 'n'. Although efficient algorithms are available [12,13,26,28], considerable computation time is required. This problem has been solved to a great extent by Prabhakara and El-Abiad [31] recently but needs further refinements to yield better stability regions. Another drawback of the method is in regard to the modelling of the system. The selection of the models for the system in Lyapunov methods is strongly limited by the practical possibility of constructing a suitable Lyapunov function [35]. Unfortunately, till now it has not been possible to include refinements like governor dynamics, saturation and saliency effects etc., into the Lyapunov functions for multimachine systems. Therefore the general problem of multimachine systems has been considered [13] only on the basis of certain simplifying assumptions. The

solution to some of these foregoing problems calls for a new approach to the problem of stability of large scale power systems. In this thesis an attempt has been made to cover some new grounds in this direction. They are

(i) Application of the concept of vector Lyapunov function for the large scale power system stability problem.

(ii) Development of new algorithms to determine the stability boundaries of multi-machine systems

and (iii) development of dynamic equivalents for power systems using the Lyapunov function concept.

We now briefly review the pertinent literature in these areas before outlining the objective and the scope of the present thesis.

1.4 THE STATE OF THE ART:

1.4.1 The vector Lyapunov function approach:

Most complex systems are made up of a number of low order interacting systems. These low order systems in hierarchial theory [36] are called 'Subsystems'. These subsystems are arrived at on the basis of decomposition of the large system. Decomposition and aggregation have long been used in economic theory and found an engineering

application [37] through the tearing techniques of Kron [38]. Today's power systems being large and complex become natural candidates for such application. The analysis of load flow and short circuit problems in large scale systems using the diakoptic method is well known. One therefore is motivated to extend these ideas to the stability problems of large scale systems using Lyapunov methods. Such a procedure was first presented by Bailey [39]. The technique consists in decomposing the system into low order subsystems. Suitable Lyapunov functions are constructed in the usual manner for these subsystems satisfying certain sign definite properties. On a higher hierarchial level, the concept of a vector Lyapunov function [40] is introduced in which these scalar functions form the elements of the vector Lyapunov function defined for the composite system. Using the properties of interactions among the subsystems conditions for the stability of the overall system are derived. These ideas, however, did not receive sufficient attention until recently when Siljak and others[41-51] developed the theory further in a manner suitable for application to physical systems. In spite of these developments applications to realistic systems are yet unknown in the literature in general and the area of the power systems in particular. Probably the reason for this is that a suitable decomposition procedure,

which is a key concept in hierarchial theory, could not be developed for implementation of the theory. In this thesis an attempt has been made to apply vector Lyapunov function to the stability analysis of large scale power systems [52].

1.4.2 Stability regions:

The problem of estimating the stability regions of multimachine power systems for Lyapunov methods was identified with the computation of the unstable equilibrium points of the post-fault system [5,6,12-23,26-28, 31] and evaluating the V-function at these points. The minimum of the values so computed yields a constant ε that gives an estimate of the stability domain defined by the inequality (1.4). It is well known that there are $(2^{n-1}-1)$ unstable equilibrium points for an n-machine system and the computation of these points becomes prohibitive when 'n' is large. In this connection special studies have been carried out in this area and also pseudo random search techniques have been proposed. Notable among these are the works of Ribbens Pavella [12], Tavora and Smith [26,27] and Uyemura et al [28]. Some interesting results based on physical energy type approximate considerations [17,20] have also been proposed and applied to estimate the stability domains. An approximate 'n-dimensional cube' method has also been proposed and used [29]. The basic philosophy of this

method consists in assuming an n-dimensional cube (n being the number of machines) around the post-fault stable equilibrium point. The minimum value of V ($\underline{X}$) on this surface is maximized with respect to the side length itself and the resulting value is used as an estimate of ε in equation (1.4). A further approximate method [30,34] is based on a series expansion around suitable known points. The computation of V ($\underline{X}$) at the corners of an (n-1) dimensional parallelopiped and choosing the minimum of these values was suggested by Murthy [24]. Recent work of Prabhakara and El-Abiad [31] approximates the unstable equilibrium points on considerations of those similar to a single machine connected to an infinite bus system. The present thesis utilizes the information associated with the nonlinearities occuring in the multimachine power system models cast in the Lure'-Popov form. These nonlinearities are known to violate the Popov sector conditions. Utilizing this property explicit expressions are obtained for the stability regions based on certain approximations and thus constitutes a generalization of a similar work due to Pai et al [9] for single machine power systems. Results of reference[31]are shown to be a special case of this general approach.

1.4.3 Dynamic Equivalents:

In this section we review the state of the art in the area of system simplification. The growth of power systems in the recent years has made stability analysis of the system in a detailed manner difficult due to the limited core memory of even the present day computers. A possible solution to this problem is to reduce the size of the system by the use of equivalents to represent portions of the network beyond the area of immediate interest. Usually in power systems, a fault in one section of the system may not significantly affect the behaviour of some other section of the system. In such cases it is not necessary to model this latter section in detail and can be replaced by an 'equivalent' representing it while studying the behaviour of the faulted section. Thus a reduced representation of the system called the 'dynamic equivalent' is available. One method of obtaining such an equivalent is by utilizing the property of 'coherency' among machines in the system.and replacing such machines in the equivalent by a single machine. Such equivalents were advocated in the early fifties when the systems at that time were too big to be manageable on the A.C. network analyzers. In this method of equivalencing, groups of machines that 'swing' together, during the period of study in the faulted section, are combined. Identification of these coherent machines

is done by conducting a detailed study on the system and applying a set of criteria [53-59]. Later a dynamic equivalent is developed by using certain other criteria [53-58,59]. Recognizing that the likelihood of machines swinging together [2] is increased by (i) a decreased impedance between the machines (ii) the initial angular position being in close proximity (iii) the inertia constants being almost equal and (iv) remoteness of the fault or the source of disturbance, Lee and Schweppe [59] developed the 'features' of pattern recognition and used them for coherency identification.

Other approaches in the development of dynamic equivalents employ modal techniques and system order reduction [60,61,62 - 64]. In this thesis Lyapunov functions are used for coherency recognition and also to obtain dynamic equivalents.

1.5 SCOPE AND OUTLINE OF THE THESIS:

The main objectives and chapterwise summary of the work reported in this thesis will now be outlined.

The main objectives of this thesis are

(i) to present an alternative technique for the stability analysis of large scale power systems using vector Lyapunov functions.

(ii) to derive explicit expressions for the stability

domains of multimachine power systems based on the sector violation criteria of multi-nonlinear systems,

and(iii) to develop an effective criterion for coherency recognition and construction of a dynamic equivalent based on Lyapunov's method.

The following is a chapterwise summary of the work being reported in this thesis.

Chapter II begins with the definitions of overdescribed models [65,66] of systems and relates these concepts to state uncontrollability. The state space model of power systems in the 2n dimensional state space is developed for a 2-machine power system and is shown to be overdescribed. Explicit dependency relationships among the state variables are derived both for uniform and nonuniform damping cases. from which the legitimate or the minimal order of the state vector is derived. The analysis is then generalized to an n-machine system. State models in the 2n and (n+m) dimensional state spaces for multimachine systems for the uniform and the nonuniform damping cases respectively are then introduced. These models are useful in decomposing the multimachine system into lower order subsystems for the application of the vector Lyapunov approach in Chapter III. These are shown to be overdescribed and the

legitimate order of the state vector is derived. It is concluded that the legitimate order of the state vector for an n-machine system is (2n-2) in the case of uniform damping and (2n-1) for the nonuniform damping case. Thus a physical insight into the reasons of uncontrollability of certain power system models is provided in this chapter.

Chapter III introduces the concept of vector Lyapunov function as applied to a large scale system stability problem. Firstly the concepts of the subsystems and their interactions are introduced by a state variable decomposition of the original dynamical system. The required conditions on the scalar functions for asymptotic stability of the individual 'free' subsystems and also those on the interactions are next reviewed. A vector Lyapunov function for the composite system is then defined and conditions for the asymptotic stability of the overall system are given by means of a theorem due to Grujic and Siljak [48]. This vector Lyapunov function is then used to construct a scalar Lyapunov function for the composite system. This latter function is useful in determining a region of stability for the overall system. The application of this theory to the power system problem is next considered. The decomposition procedure of the system is outlined for a 3-machine case first and is later generalized to an n-machine system. It is shown that

each of these subsystems is described in its minimal state space. Lure' type Lyapunov functions are constructed for each of them. Conditions for stability of the post-fault composite power system are later derived via a vector Lyapunov function. A scalar Lyapunov function for the composite system is then constructed using the vector Lyapunov function to estimate the stability region. The entire procedure is finally illustrated with reference to a 3-machine example.

Chapter IV is devoted to the estimation of stability domains for multinonlinear systems cast in the Lure' -Popov form in which the nonlinearities violate the Popov's sector condition. Using this information an expression is derived for the stability domains constituting a generalization of the result for a system with a single nonlinearity [67,68]. An application of this result to the power system stability problem is next discussed. For this purpose, the problem of obtaining the stability regions for multimachine system is formulated based on the sector violation properties of the power system nonlinearities. The method of application of the above mentioned result is then described. An explicit expression for the stability region is also obtained. It is also shown in this chapter that a method recently described by Prabhakara and El-Abiad [31] for estimating the unstable

equilibrium point can be brought into the framework of the present analysis, thus providing a theoretical basis to their method. An alternate procedure that provides better stability regions is also proposed in this chapter. Finally these methods are implemented on a 5-machine system and the critical clearing times obtained by these methods are compared with that determined by direct simulation.

The application of Lyapunov functions for coherency identification in a multimachine power system is demonstrated in chapter V. Using the kinetic and the potential energy components of an energy type Lyapunov function a new definition of coherency is proposed. This definition is them used for both coherency identification and development of a dynamic equivalent. An algorithm for coherency recognition on the basis of the new definition is presented. Later, explicit expressions for the self admittance of the equivalent generator replacing a coherent group and the transfer admittances between this equivalent machine and the rest of the machines are derived. The coherency recognition algorithm is implemented on a 44 bus 15 generator system. After obtaining the coherent groups, the dynamic equivalent is developed. Swing curves of the retained machines are obtained by conducting a stability study using this equivalent and are compared with those obtained

from a detailed study. Use of these dynamic equivalents in the study of large scale systems via Lyapunov methods is discussed.

The concluding sixth chapter highlights the major contributions of this thesis and certain problems encountered during the course of this research are presented. Suggestions for future work are then given.

CHAPTER II

STATE OVERDESCRIPTION AND UNCONTROLLABILITY IN POWER SYSTEM MODELS

2.1 INTRODUCTION:

This chapter deals with the development of state space models for multimachine power systems for use in transient stability analysis. A number of state models have been cited in the literature [7,10,12,14,15,16] with the result that there was a great deal of controversy regarding the legitimate order of the system to be used in the systematic construction of Lyapunov functions. This controversy has, however, been settled either through the control theoretic concept of minimal realization [69] or by physical arguments as suggested by Ribbens Pavella [11,12]. It is by now well established that for constant mechanical inputs and constant flux linkages, a uniformly damped and an undamped n-machine system should be treated in a (2n-2) dimensional state space and the non-uniform damping case of the system in the (2n-1) dimensional state space. The question of studying all these cases in a unified framework for the sake of comparison and convenience was analysed in depth by Willems [21] by defining stability not as a state stability concept but rather an output stability property or what is called partial stability.

Although certain criteria exist for testing the controllability of the system, little insight into the

exact mechanism which causes this uncontrollability in certain of the state models is provided by these techniques. There, in fact, appears to be a lacuna in justifying the choice of the state variables while obtaining the state description of the power system differential equations. The presence of uncontrollability is normally associated with the inefficiency in the manner in which the effect of the control propogates through the system [70]. When the swing equations are put in the form of a set of first order differential equations, the power angle nonlinearities constitute the 'Control variables', in the language of modern control theory. Controllability as a physical concept is therefore not meaningful although it has been applied in a mathematical sense, rather mechanically [69]. In this chapter we take a view point following Johnson [65] in connection with overdescribed system models. It will be shown that some of the power system models are, in fact, overdescribed resulting in a functional dependency of some of the state variables on others. It is this fact that results in the mathematical 'Uncontrollability' of the system. The process of obtaining a legitimate order of the state vector is also described and this order is found to be identical to that obtained in the literature [69]. Such a realization is indeed a prerequisite for applying the results of modern stability theory.

2.2 UNCONTROLLABILITY AND OVERDESCRIPTION:

The notion of controllability of a linear dynamical system has been used extensively in the control literature and is a property associated with the effect of control forces on the state of the system. The following definition of controllability is due to Kalman [71].

DEFINITION 1: A system is said to be controllable if it is possible to find a control vector $\underline{U}(t)$ which in a specified time t_f will transfer the system between two arbitrarily specified finite states $\underline{X}_0$ and $\underline{X}_f$.

Clearly, a system is said to be uncontrollable if it is not controllable. It is necessary to ascertain this property of the linear dynamical system before one can apply optimal control theory. However if the system is uncontrollable, reasons for such uncontrollability have not been fully explained. Recent work of Johnson [65] explains a particular mode of uncontrollability arising due to 'System Overdescription'. We thus have the following definition.

DEFINITION 2: A system described by

$$\dot{\underline{X}} = \underline{F}\,(\underline{X},\underline{U}) \tag{2.1}$$

where $\underline{X}$ is an N-dimensional state vector and $\underline{U}$ is an r-dimensional control vector, is said to be overdescribed if its state vector of order N has components which are functionally dependent and $N \geq p$ where 'p' is the legitimate

order of the state vector.

DEFINITION 3: A state vector $\underline{X}$ is considered to be of legitimate order if it contains the minimal number of state variables which are functionally independent to describe the physical system.

Definition (2) implies that there is a redundancy in the state variable description which may not be explicitly exhibited in the system (2.1). In such a case there always exists a set of functionally independent algebraic functions $\{\xi_i(\underline{X})\}$, i=1,2,..(N-p) such that

$$\xi_j(X_1, X_2, \ldots X_N) = \text{constant} \quad , \; j=1,2,\ldots(N-p). \tag{2.2}$$

Although these equations are not known a priori, the effect of these is manifested in (2.1). Hence the right hand sides of (2.1) should satisfy the following set of relations:

$$\left.\begin{aligned} &\varphi_{11}(\underline{X})\; F_1(\underline{X},\underline{U}) + \varphi_{12}(\underline{X})\; F_2(\underline{X},\underline{U}) + \ldots \\ &\qquad\qquad + \varphi_{1N}(\underline{X})\; F_N(\underline{X},\underline{U}) = 0 \\ &\;\vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ &\varphi_{(N-p),1}(\underline{X})\; F_1(\underline{X},\underline{U}) + \varphi_{(N-p),2}(\underline{X})\; F_2(\underline{X},\underline{U}) + \ldots \\ &\qquad\qquad + \varphi_{(N-p),N}(\underline{X})\; F_N(\underline{X},\underline{U}) = 0 \end{aligned}\right\} \tag{2.3}$$

where the F_i's are the components of $\underline{F}(\underline{X},\underline{U})$. This set of equations is derived by differentiating (2.2) which gives

$$\sum_{i=1}^{N} \frac{\partial \xi_j(\underline{X})}{\partial X_i} \dot{X}_i = 0 \quad , \quad j=1,2,\ldots(N-p) \; . \tag{2.4}$$

Define

$$\frac{\partial \xi_j(\underline{X})}{\partial X_i} = \varphi_{ji}(\underline{X}) \; . \tag{2.5}$$

Then with (2.1) and (2.5), equations (2.4) yield

$$\sum_{i=1}^{N} \varphi_{ji}(\underline{X}) \; F_i(\underline{X}) = 0 \quad , \quad j=1,2,\ldots(N-p) \tag{2.6}$$

Hence (2.3) follows from (2.6).

Again the φ_{ji}'s are not known a priori. These coefficients $\varphi_{ji}(\underline{X})$ form a set of Pfaffian forms of the type

$$\varphi_{j1}(\underline{X})dX_1 + \varphi_{j2}(\underline{X})dX_2 + \cdot \cdot \cdot + \varphi_{jN}(\underline{X})dX_N = 0, \quad j=1,2,\ldots(N-p). \tag{2.7}$$

If these are integrable, then, the required functionally dependent relations among the state variables are obtained. We are thus led to the following theorem.

THEOREM 2.1: [65]

The mathematical model (2.1) is uncontrollable if there exists a positive number $p < N$ and a set of coefficients $\varphi_{ji}(\underline{X})$, $j=1,2,\ldots(N-p)$, and $i=1,2,\ldots N$, not all zero, such that, the following conditions are satisfied:

(i) the right hand sides of (2.1) satisfy (2.3) for all $\underline{X}$ and for all $\underline{U}$

and (ii) the coefficients $\varphi_{ji}(\underline{X})$ form a set of Pfaffian

forms that are integrable.

In that case the uncontrollability is due to the state overdescription in the sense of definition (2).

A NOTE ON PFAFFIAN FORMS:

A single expression of the type

$$\varphi_{11}(\underline{X})dX_1 + \varphi_{12}(\underline{X})dX_2 + \ldots + \varphi_{1N}(\underline{X})dX_N = 0 \qquad (2.8a)$$

is called a Pfaffian form. The Pfaffian form is said to be integrable if and only if there exists a scalar function

$$\xi(\underline{X}) = \text{constant} \qquad (2.8b)$$

such that the total derivative $d\xi$ is proportional to (2.8a). In other words

$$\frac{d\xi}{dX_i} = \pi(\underline{X})\varphi_{1i}(\underline{X}) \qquad (2.8c)$$

where $\pi(\underline{X})$ is a common factor of proportionality. For cases where $\varphi_{1i}(\underline{X})$ are constants, (2.8a) is always integrable [65].

2.3 MATHEMATICAL MODEL OF THE POWER SYSTEM:

The following simplifying assumptions are made in the transient stability of power systems:

(i) The voltage behind the transient reactance is assumed constant. In other words it is assumed that the flux linkages of the various machines are constant. This assumption is generally valid for the first swing stability analysis.

(ii) Damping is assumed to be directly proportional to the slip velocity and is thus mainly due to the mechanical friction and asynchronous torques.

(iii) The mechanical power inputs to the synchronous machines are assumed constant during the transient period. This is a satisfactory assumption because the time constants of the governors are much larger than the duration of the transient swings.

(iv) Transfer conductances are neglected.

(v) Saturation is neglected.

(vi) Only round rotor machines are considered thus neglecting the effect of saliency. Machine resistances are neglected.

Under the above assumptions the differential equations that describedthe dynamics of an n-machine power system are given by

$$M_i \frac{d^2\delta_i}{dt^2} + D_i \frac{d\delta_i}{dt} = P_{mi} - P_{ei} , \quad i=1,2,..n. \tag{2.9}$$

The electrical power output P_{ei} of machine i is given by

$$P_{ei} = E_i^2 G_{ii} - \sum_{\substack{j=1 \\ j\neq i}}^{n} E_i E_j B_{ij} \sin(\delta_i - \delta_j) . \tag{2.10}$$

Substitution of (2.10) into (2.9) yields

$$M_i \frac{d^2\delta_i}{dt^2} + D_i \frac{d\delta_i}{dt} = P_i - \sum_{\substack{j=1 \\ j\neq i}}^{n} E_i E_j B_{ij} \sin(\delta_i - \delta_j) \quad (2.11)$$

where $P_i = P_{mi} - E_i^2 G_{ii}$. (2.12)

The equilibrium states of the system (2.10) are obtained by setting both $\dot{\delta}_i$ and $\ddot{\delta}_i$ equal to zero resulting in

$$P_i = \sum_{\substack{j=1 \\ j\neq i}}^{n} E_i E_j B_{ij} \sin(\delta_i - \delta_j) \ . \quad (2.13)$$

Although there are n equations in (2.13) with (n-1) angular difference variables these equations are not overdetermined. In fact, (2.13) suggests that

$$\sum_{i=1}^{n} P_i = \sum_{i=1}^{n} (P_{mi} - E_i^2 G_{ii}) = 0 \quad (2.14)$$

is a necessary condition for the existence of an equilibrium state [14]. This is logical since equations (2.14) specify the real power balance in the system. Assuming (2.14) to hold, if all the terms in (2.13) refer to the post-fault system, then the post-fault rotor angle δ_i^o, i=1,2,...n satisfy (2.13) identically. i.e.

$$P_i - \sum_{\substack{j=1 \\ j\neq i}}^{n} E_i E_j B_{ij} \sin(\delta_i^o - \delta_j^o) = 0 \ , \qquad i = 1,2,\ldots n. \quad (2.15)$$

Then equations (2.11) can be written in the form

$$M_i \frac{d^2\delta_i}{dt^2} + D_i \frac{d\delta_i}{dt} = \sum_{\substack{j=1\\j\neq i}}^{n} E_i\ E_j\ B_{ij}\ \sin(\delta_i^o - \delta_j^o)$$

$$- \sum_{\substack{j=1\\j\neq i}}^{n} E_i\ E_j\ B_{ij}\ \sin(\delta_i - \delta_j)$$

$$= - \sum_{\substack{j=1\\j\neq i}}^{n} E_i\ E_j\ B_{ij}\ [\sin(\delta_i - \delta_j) - \sin(\delta_i^o - \delta_j^o)] \qquad (2.16)$$

Equations (2.16) can be cast into a compact vector matrix differential equation of the form

$$\left.\begin{aligned} \dot{\underline{X}} &= A\underline{X} - B\ \underline{f}(\underline{\sigma}) \\ \underline{\sigma} &= C^T\underline{X} \end{aligned}\right\} \qquad (2.17)$$

where the state vector $\underline{X}$ is of the proper order depending on the choice of the state variables. So too, are the matrices A,B and C. The vector $\underline{\sigma}$ is an m (= n(n-1)/2) dimensional vector and $\underline{f}(\underline{\sigma})$ is a vector valued function with m components whose i^{th} component depends on the i^{th} component of the output vector $\underline{\sigma}$. Much of the earlier controversy centered around the dimensionality of the state vector $\underline{X}$ and the choice of the state variables with the matrices A,B and C dependent on the way the state variables are selected. We shall now consider the different forms of the state representations discussed in the literature and verify whether or not the choice of the corresponding state vector results in an overdescription

of the dynamical system. The case of a 2-machine system will be considered first and later generalized to an n-machine system. Explicit relationship existing between the states of the system due to overdescription will be obtained which in fact results in the so called 'Uncontrollability' of the mathematical model. It may be emphasized once again that overdescription implies a redundancy in the description of the state variables leading to a functional dependency among these variables.

2.4 OVERDESCRIPTION, REDUNDANCY AND UNCONTROLLABILITY:

2.4.1 Two machine case:

The following choice of the state variable was initially found to be an obvious one [14] to put the system description in the form (2.17)

$$\begin{aligned} X_1 &= \omega_1 = \dot{\delta}_1 \quad ; \quad X_3 = \delta_1 - \delta_1^o \\ X_2 &= \omega_2 = \dot{\delta}_2 \quad ; \quad X_4 = \delta_2 - \delta_2^o \end{aligned} \tag{2.18}$$

The state equations of the 2-machine system from (2.16) and (2.17) with n=2 are given by

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} -\lambda_1 & 0 & 0 & 0 \\ 0 & -\lambda_2 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} - \begin{bmatrix} 1/M_1 \\ -1/M_2 \\ 0 \\ 0 \end{bmatrix} f(\sigma) \tag{2.19}$$

$$\sigma = [\ 0\ 0\ 1\ -1] \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

where $\quad f(\sigma) = E_1\ E_2\ B_{12}\ [\sin(\sigma + (\delta_1^o - \delta_2^o)) - \sin(\delta_1^o - \delta_2^o)]$

The two machine system is thus cast in the 2n (=4) dimensional state space.

Equations (2.19) can be put in the compact form

$$\dot{\underline{X}} = \underline{F}(\underline{X}, \underline{f}) \tag{2.20a}$$

where $\underline{F}$ is a 2n-dimensional vector function, the elements of which are given by the right hand sides of (2.19).

Thus

$$\begin{aligned} F_1(\underline{X}) &= -\lambda_1 X_1 - \frac{1}{M_1} f(\sigma) \\ F_2(\underline{X}) &= -\lambda_2 X_2 + \frac{1}{M_2} f(\sigma) \\ F_3(\underline{X}) &= X_1 \end{aligned} \tag{2.20b}$$

and $\quad F_4(\underline{X}) = X_2$.

If the system is overdescribed, this set of equations should satisfy a set of simultaneous linear algebraic relations of the type (2.3) and given by

$$\varphi_{j1}(\underline{X})F_1(\underline{X}) + \varphi_{j2}(\underline{X})F_2(\underline{X}) + \dots + \varphi_{j4}(\underline{X})F_4(\underline{X}) = 0,$$

$$j = 1,2,..(4-p) \tag{2.21}$$

where 'p' is the unknown but legitimate order of the state vector $\underline{X}$.

Substituting (2.20) into (2.21) we obtain

$$\varphi_{j1}(\underline{X})\{-\lambda_1 X_1-(1/M_1)f(\sigma)\} + \varphi_{j2}(\underline{X})\{-\lambda_2 X_2+(1/M_2)f(\sigma)\}$$
$$+ \varphi_{j3}(\underline{X})\, X_1 + \varphi_{j4}(\underline{X})\, X_2 = 0, \quad j=1,2,\ldots (N-p). \quad (2.22)$$

It is found that only one set of φ_{ji}'s, i=1,2,...4 satisfying equation (2.22) is possible. This set is given by

$$\varphi_{j1}(\underline{X}) = M_1 \; ; \; \varphi_{j2}(\underline{X}) = M_2 \; ; \; \varphi_{j3}(\underline{X}) = D_1 \text{ and } \varphi_{j4}(\underline{X}) = D_2 \quad (2.23)$$

These φ_{ji}'s form an integrable Pfaffian form given by

$$M_1 dX_1 + M_2 dX_2 + D_1 dX_3 + D_2 dX_4 = 0 \,. \quad (2.24)$$

This implies j=1 and hence p = 4-1=3 is the legitimate order. Integration of (2.24) yields

$$M_1 X_1 + M_2 X_2 + D_1 X_3 + D_2 X_4 = K \quad (2.25)$$

where K is a constant. This is a functional dependency relationship among the state variables of $\underline{X}$. This in a geometric sense implies that, the set of all the physical states $\{X_1, X_2, X_3, X_4\}$ lie on a 3-dimensional manifold M in a 4-dimensional state space as shown in Figure 2.1. States of the system not on M do not physically exist and hence cannot be reached by the physical system irrespective of the way in which initial states are chosen. This, in the mathematical sense results in uncontrollability of the physical system. Application of the standard criteria for controllability to (2.19) will also reveal it to be

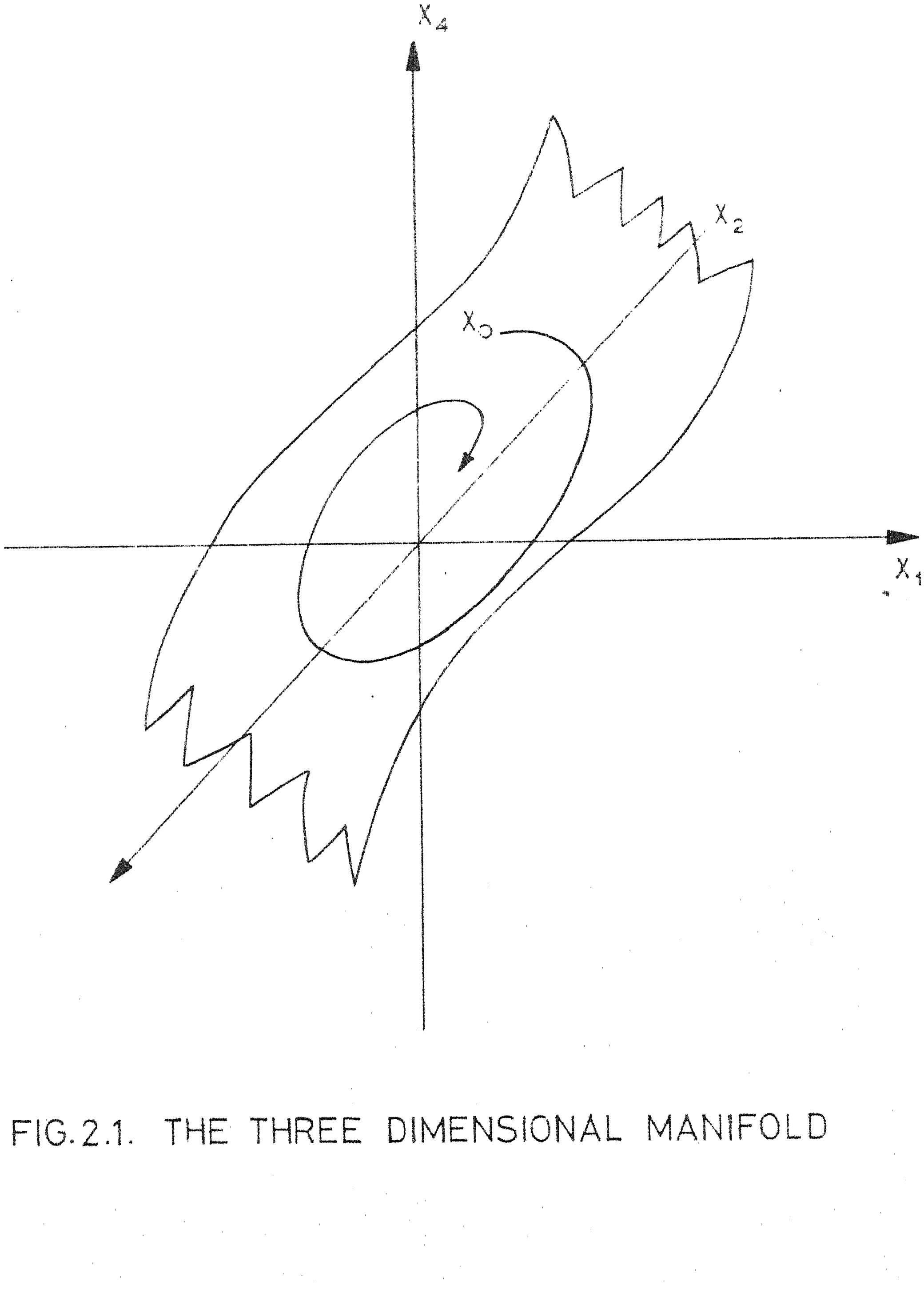

FIG. 2.1. THE THREE DIMENSIONAL MANIFOLD

uncontrollable. It is therefore necessary to redefine the state variables. Two different choices are possible (i) relative rotor velocities and (ii) relative rotor angles. At this juncture two different cases of the system arise. They are

(i) Non-uniform damping where $\lambda_1 \neq \lambda_2$

and (ii) uniform damping where $\lambda_1 = \lambda_2$.

We shall consider each of these cases separately.

Case (i): Non-uniform damping:

Equations (2.9) show that the state model cannot be constructed with relative rotor velocities as the state variables thus leaving relative rotor angles to be considered for state representation. Thus the legitimate state vector has components given by

$$X_1 = \omega_1$$

$$X_2 = \omega_2$$

and $X_3 = (\delta_1 - \delta_2) - (\delta_1^o - \delta_2^o)$. (2.26)

With this set of variables, the state space model is of the form

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} -\lambda_1 & 0 & 0 \\ 0 & -\lambda_2 & 0 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} - \begin{bmatrix} 1/M_1 \\ -1/M_2 \\ 0 \end{bmatrix} f(\sigma)$$

$$\sigma = [0 \quad 0 \quad 1] \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} \tag{2.27}$$

It is easily verified in a manner outlined in the previous section that no nontrivial φ_{ji}'s can be found for the system (2.27) satisfying (2.3). Thus the legitimate order is indeed 3. Also, the usual controllability conditions reveal that the system is completely controllable. The transfer function $W_N(s)$ of the linear part of the system is given by

$$W_N(s) = \frac{\frac{1}{M_1}(s+\lambda_2) + \frac{1}{M_2}(s+\lambda_1)}{s(s+\lambda_1)(s+\lambda_2)} \tag{2.28}$$

Subscript N refers to the non-uniform damping case.

Case (ii): Uniform damping:

By setting $\lambda_1 = \lambda_2 = \lambda$ in equation (2.27) one obtains a system description of the form

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} -\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} - \begin{bmatrix} 1/M_1 \\ -1/M_2 \\ 0 \end{bmatrix} f(\sigma)$$

$$\sigma = [0 \quad 0 \quad 1] \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} . \tag{2.29}$$

For these equations no separate set of φ_{ji}'s can be obtained that satisfy (2.3). Hence the system is not overdescribed in the sense of Johnson [65]. But from (2.29), one can trivially obtain a relationship of the type

$$M_1\dot{X}_1 + M_2\dot{X}_2 = -\lambda(M_1X_1 + M_2X_2) \tag{2.30}$$

which can be easily integrated to give

$$M_1X_1 + M_2X_2 = \{M_1X_1(0) + M_2X_2(0)\}\ e^{-\lambda t} \tag{2.31}$$

where $X_1(0)$ and $X_2(0)$ are the initial values of the states X_1 and X_2. This exhibits a clear dependency among the variables. This sort of physical dependency relationship among the state variables was pointed out by Rosenbrock [66] exhibiting thus a clear redundancy in the state variables. Johnson's conditions fail to show this sort of overdescription because theorem 1 is only a necessary condition for overdescribed systems. It is also observed that this pehnomenon manifests itself in pole zero cancellations in the transfer function $W_U(s)$ of the system. From equations (2.28), by setting $\lambda_1 = \lambda_2 = \lambda$ it is noticed that there is a pole zero cancellation. Thus

$$W_U(s) = \frac{\frac{1}{M_1} + \frac{1}{M_2}}{s(s+\lambda)} \quad . \tag{2.32}$$

Subscript U refers to the uniform damping case.
This redundancy again calls for a redefinition of the state variables. The relative velocity $(\omega_1 - \omega_2)$ is chosen as one of the state variables instead of ω_1 and ω_2. Thus with relative velocity and relative rotor angle as the state variables, the state vector is defined by

$$\begin{aligned} X_1 &= \omega_1 - \omega_2 \\ X_2 &= (\delta_1 - \delta_2) - (\delta_1^o - \delta_2^o). \end{aligned} \tag{2.33a}$$

Equations (2.9) can be recast in the form

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} -\lambda & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} - \begin{bmatrix} 1/M_1 + 1/M_2 \\ 0 \end{bmatrix} f(\sigma)$$

$$\sigma = [0 \quad 1] \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} . \tag{2.33b}$$

$f(\sigma)$ is the same as in equation (2.19). It is easily verified that (2.33) is completely controllable and hence the order of the legitimate state vector in this case is 2.

REMARK: The state models derived in equations (2.29) and (2.33) are useful in constructing Lyapunov functions for the subsystems in Chapter III which deals with vector Lyapunov functions.

We now generalize these results for an n-machine power system described in the 2n-dimensional state space [7,8,14,15].

2.4.2 n-machine case:

If we define the state variables by

$$\left.\begin{aligned} X_i &= \omega_i \\ X_{i+n} &= (\delta_i - \delta_i^o) \end{aligned}\right\} , \quad i = 1,2,\ldots n. \tag{2.34}$$

the state model is then given by

$$\begin{aligned} \dot{\underline{X}} &= A^* \underline{X} - \underline{B}^* \underline{f}(\underline{\sigma}) \\ \underline{\sigma} &= C^{*T} \underline{X} \end{aligned} \tag{2.35}$$

where

$$A^* = \begin{bmatrix} -\Lambda_{nn} & 0_{nn} \\ I_{nn} & 0_{nn} \end{bmatrix} ; \quad B^* = \begin{bmatrix} M^{-1} K_{nm} \\ 0_{nm} \end{bmatrix} \text{ and } C^{*T} = [0_{mn} \quad K_{nm}^T]$$

in which $\Lambda = \text{diag}(\lambda_i)$, $i = 1,2,\ldots n$, $M = \text{diag}(M_i)$, $i=1,2,\ldots n$. 0_{nn} and I_{nn} are the null and unit matrices of order n respectively. The matrix K_{nm} $(m = n(n-1)/2)$ is given by [7]

$$K_{nm} = [K_1 \,\vdots\, K_2 \,\vdots\, \ldots \,\vdots\, K_{n-1}] \tag{2.36}$$

where K_i, $i = 1,2,\ldots(n-1)$ is an $n\times(n-i)$ matrix having the following structure [7]

$$K_i = \begin{bmatrix} & 0_{(i-1)(n-i)} & & \\ \hline 1 & 1 & \ldots & 1 \\ -1 & 0 & \ldots & 0 \\ 0 & -1 & \ldots & 0 \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \ldots & -1 \end{bmatrix} \tag{2.37}$$

Double subscript notation in equations (2.35) - (2.37) is used to identify the rows and columns of the respective matrices. The function $\underline{f}(\underline{\sigma})$ is an m-vector whose ith component is a function of the ith component of $\underline{\sigma}$ only. Thus $f_i(\sigma_i)$ is given by

$$f_i(\sigma_i) = E_k\ E_j\ B_{kj}[\sin(\sigma_i + \theta_i) - \sin\theta_i],\ i = 1,2,\ldots m \tag{2.38}$$

where $\theta = K^T_{nm}\ \underline{\delta}^o$ and $\underline{\delta}^o$ is an n-vector given by

$$\underline{\delta}^o = [\delta^o_1, \delta^o_2, \ldots\ \delta^o_n]^T. \tag{2.39}$$

It has been shown by Murthy [24] that the system (2.35) is uncontrollable and unobservable. Choose the φ_{ji}'s such that the N-vector (here N = 2n) $\underline{\tilde{\Phi}}$ constituting the φ_{ji}'s as its elements and of the form

$$[\varphi_{j1}, \varphi_{j2}, \ldots\ \varphi_{jN}]^T \tag{2.40}$$

is given by

$$\underline{\tilde{\Phi}} = [M_1, M_2, \ldots\ M_n, D_1, D_2, \ldots\ D_n]^T . \tag{2.41}$$

It can now be shown that

$$\underline{F}^T\ \underline{\tilde{\Phi}} = 0 \tag{2.42}$$

where $\underline{F}$ is a vector of the functions on the right hand sides of (2.35). These φ_{ji}'s yield an integrable Pfaffian forms of the type

$$\sum_{i=1}^{n} M_i dX_i + \sum_{i=1}^{n} D_i dX_{n+i} = 0 . \tag{2.43}$$

On integration (2.43) results in

$$\sum_{i=1}^{n} (M_i X_i + D_i X_{n+i}) = \text{constant} \tag{2.44}$$

thereby concluding that the system is overdescribed in this state space. For the non-uniform damping case, (2.44) is the only functional dependency relationship and hence the legitimate order 'p' of the state vector $\underline{X}$ is given by

$$p = (2n - 1). \tag{2.45}$$

For the uniform damping case, however, the order of the statevector can be reduced further by 1 in view of the relation

$$\sum_{i=1}^{n} M_i X_i = \left(\sum_{i=1}^{n} M_i X_i(0)\ e^{-\lambda t} \right) \tag{2.46}$$

which again exhibits a redundancy in the state variable description according to Rosenbrock [66]. Hence the legitimate order of the state vector is (2n-2) for the uniform damping case.

2.5 ALTERNATE FORMS OF n-MACHINE MODELS:

We shall next consider two particular state models in the 2m ($m = \frac{n}{2}(n-1)$) and (m+n) state spaces for the uniform damping and non-uniform damping cases

respectively. In view of the discussion in Sec.2.4 regarding the legitimate system order, these models are overdescribed. However, their consideration is merited by the fact that these models lend themselves for decomposition into subsystems for applying vector Lypunov functions. Together with the description of these models we briefly indicate the nature of overdescription.

2.5.2 System description in the 2m-dimensional state space [12]: (Uniform damping only)

The state variables chosen to represent the uniformly damped case [12] in the 2m(=n(n-1)) dimensional state space are given by

$$\left.\begin{aligned} X_i &= \omega_k - \omega_j \\ X_{i+m} &= (\delta_k-\delta_j) - (\delta_k^o-\delta_j^o) \end{aligned}\right\} ,(i=1,2,\ldots m \text{ and } k,j = 1,2,\ldots n, k<j) \tag{2.47}$$

The state space model is then given by

$$\begin{aligned} \dot{\underline{X}} &= A\,\underline{X} - B\,\underline{f}(\underline{\sigma}) \\ \underline{\sigma} &= C^T\,\underline{X} \end{aligned} \tag{2.48}$$

where

$$A = \begin{bmatrix} -\Lambda_{mm} & 0_{mm} \\ I_{mm} & 0_{mm} \end{bmatrix}; \quad B = \begin{bmatrix} K_{nm}^T\,M^{-1}\,K_{nm} \\ 0_{mm} \end{bmatrix}$$

and $C^T = [0_{mm} \quad I_{mm}]$.

The $f_i(\sigma_i)$'s are the same as in equation (2.38). Over-description in these models will now be illustrated with reference to a 3-machine example.

3-Machine case:

Consider a 3-machine system with uniform damping. The state variables according to equation (2.47) are

$$
\begin{aligned}
X_1 &= (\omega_1 - \omega_2) \quad ; \quad X_4 = (\delta_1-\delta_2) - (\delta_1^0-\delta_2^0) \\
X_2 &= (\omega_1 - \omega_3) \quad ; \quad X_5 = (\delta_1-\delta_3) - (\delta_1^0-\delta_3^0) \\
X_3 &= (\omega_2 - \omega_3) \quad ; \quad X_6 = (\delta_2-\delta_3) - (\delta_2^0-\delta_3^0)
\end{aligned}
\tag{2.49}
$$

since 2m is 6. The state model is given by

$$
\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \cdot \\ \cdot \\ \cdot \\ \dot{X}_6 \end{bmatrix}
=
\begin{bmatrix}
-\lambda & 0 & 0 & 0 & 0 & 0 \\
0 & -\lambda & 0 & 0 & 0 & 0 \\
0 & 0 & -\lambda & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ \cdot \\ X_6 \end{bmatrix}
-
\begin{bmatrix}
\frac{1}{M_1} + \frac{1}{M_2} & \frac{1}{M_1} & -\frac{1}{M_2} \\
\frac{1}{M_1} & \frac{1}{M_2} + \frac{1}{M_3} & \frac{1}{M_3} \\
-\frac{1}{M_2} & \frac{1}{M_3} & \frac{1}{M_2} + \frac{1}{M_3} \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\underline{f}(\underline{\sigma})
$$

and

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\end{bmatrix} = \begin{bmatrix}0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix} \begin{bmatrix}X_1\\ X_2\\ \vdots\\ X_6\end{bmatrix}. \qquad (2.50)$$

The elements $f_i(\sigma_i)$, $i = 1,2,3$ of the vector valued function $\underline{f}(\underline{\sigma})$ are given by

$$f_1(\sigma_1) = E_1\ E_2\ B_{12}[\sin(\sigma_1 + (\delta_1^o-\delta_2^o)) - \sin(\delta_1^o-\delta_2^o)]$$

$$f_2(\sigma_2) = E_1\ E_3\ B_{13}[\sin(\sigma_2 + (\delta_1^o-\delta_3^o)) - \sin(\delta_1^o-\delta_3^o)]$$

and

$$f_3(\sigma_3) = E_2\ E_3\ B_{23}[\sin(\sigma_3 + (\delta_2^o-\delta_3^o)) - \sin(\delta_2^o-\delta_3^o)]. \qquad (2.51)$$

The functional dependency among the state variables is apparent in their description. The dependency relations are given by

$$\begin{aligned} X_4 - X_5 + X_6 &= \text{Constant}\\ X_1 - X_2 + X_3 &= \text{Constant} \end{aligned} \qquad (2.52)$$

These two relationships being functionally independent [65], the legitimate order of the state vector is 6-2 = 4, which agrees with our earlier observation that it should be (2n-2). These results can be easily generalised to the n-machine case.

2.5.3 State model in the (n+m) dimensional state space: (For non-uniform damping only)

In this section a state model in the $\frac{n}{2}(n+1)$ (=n+m) dimensional state space for the n-machine case with non-uniform damping is proposed. The state variables chosen for this purpose are

$$X_i = \omega_i \quad , \quad i = 1,2,\ldots n$$

$$X_{i+n} = (\delta_k - \delta_j) - (\delta_k^o - \delta_j^o), \quad (i=1,2,\ldots m;\ k,j=1,2,\ldots n \quad k < j) \tag{2.53}$$

With these variables the state model is given by

$$\begin{aligned} \dot{\underline{X}} &= \tilde{A}\ \underline{X} - \tilde{B}\ \underline{f}(\underline{\sigma}) \\ \underline{\sigma} &= \tilde{C}^T\ \underline{X} \end{aligned} \tag{2.54}$$

where

$$\tilde{A} = \begin{bmatrix} -\boldsymbol{A}_{nn} & 0_{nm} \\ K_{nm}^T & 0_{mm} \end{bmatrix}; \quad \tilde{B} = \begin{bmatrix} M^{-1}\ K_{nm} \\ 0_{mm} \end{bmatrix} \text{ and } \tilde{C} = [0_{mn} \quad I_{mm}].$$

The $f_i(\sigma_i)$'s are the same as in equation (2.38). We now exhibit overdescription in the above model by considering a 3-machine example.

3-machine case:

As an illustration consider again a 3-machine system with non-uniform damping. According to (2.53), the state variable description is of the form:

$$X_1 = \omega_1 \quad ; \quad X_4 = (\delta_1-\delta_2) - (\delta_1^o-\delta_2^o)$$
$$X_2 = \omega_2 \quad ; \quad X_5 = (\delta_1-\delta_3) - (\delta_1^o-\delta_3^o) \qquad (2.55)$$
$$X_3 = \omega_3 \quad ; \quad X_6 = (\delta_2-\delta_3) - (\delta_2^o-\delta_3^o)$$

With these variables the state model is given by

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \cdot \\ \cdot \\ \cdot \\ \dot{X}_6 \end{bmatrix} = \begin{bmatrix} -\lambda_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\lambda_3 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ \cdot \\ X_6 \end{bmatrix}$$

$$- \begin{bmatrix} \frac{1}{M_1} & \frac{1}{M_1} & 0 \\ -\frac{1}{M_2} & 0 & \frac{1}{M_2} \\ 0 & -\frac{1}{M_3} & -\frac{1}{M_3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} f_1(\sigma_1) \\ f_2(\sigma_2) \\ f_3(\sigma_3) \end{bmatrix}$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ \vdots \\ X_6 \end{bmatrix} \qquad (2.56)$$

The $f_i(\sigma_i)$'s , $i = 1,2,3$ are the same as in equation (2.51).

The linear dependency among the variables X_4, X_5 and X_6 is evident. Therefore, the relationship between them can be directly written as

$$X_4 - X_5 + X_6 = 0 \tag{2.57}$$

Also no other functionally independent relationship among the state variables is found. Hence the legitimate order of the state vector $\underline{X}$ is (6-1) = 5. This agrees with the earlier conclusion that the minimal order is (2n-1) in the non-uniformly damped case.

REMARK: Although the system models derived in Sec. 2.5.2 and Sec. 2.5.3 are overdescribed, they are, nevertheless, useful in obtaining a suitable decomposition of the system (2.1) into lower order subsystems as detailed in Chapter III.

2.6 MINIMAL ORDER REPRESENTATION OF AN n-MACHINE SYSTEM

In this section the state models of an n-machine system with the state vector $\underline{X}$ of the minimal order [10] [(2n-2) for the uniform and (2n-1) for the non-uniform damping case] will be presented for the sake of completeness. These models are completely controllable and completely observable.

Case (i) Uniform damping:

The following state variables are chosen in this case with machine 1 as reference [10,69].

$$\left.\begin{aligned} X_i &= \omega_1 - \omega_i \\ X_{i+n-1} &= (\delta_1 - \delta_i) - (\delta_1^o - \delta_i^o) \end{aligned}\right\}, \quad i = 2,3,\ldots n \tag{2.58}$$

With these components the state model is

$$\begin{aligned} \dot{\underline{X}} &= A_U \, \underline{X} - B_U \, \underline{f}(\underline{\sigma}) \\ \underline{\sigma} &= C_U^T \, \underline{X} \end{aligned} \tag{2.59}$$

where

$$A_U = \begin{bmatrix} -\lambda I_{(n-1)(n-1)} & 0_{(n-1)(n-1)} \\ I_{(n-1)(n-1)} & 0_{(n-1)(n-1)} \end{bmatrix}; \quad B_U = \begin{bmatrix} K^T_{n(n-1)} M^{-1} K_{nm} \\ 0_{(n-1)m} \end{bmatrix};$$

$$C_U^T = [0_{m(n-1)} \quad J_{m(n-1)}]; \quad J_{m(n-1)} = \begin{bmatrix} I_{(n-1)(n-1)} \\ -K^T_{(n-1)(m-n+1)} \end{bmatrix} \text{ and}$$

$$K_{n(n-1)} = \begin{bmatrix} 1_{1(n-1)} \\ -I_{(n-1)(n-1)} \end{bmatrix}.$$

The row vector $1_{1(n-1)}$ has (n-1) elements each equal to unity. The $f_i(\sigma_i)$'s are the same as in equation (2.38). The transfer function matrix $W_U(s)$ of the linear part of the system is given by

$$W_U(s) = \frac{1}{s(s+\lambda)} K^T_{nm} \, M^{-1} \, K_{nm} . \tag{2.60}$$

Case (ii) Non-uniform damping:

Here the state variables chosen are [69]

$$X_i = \omega_i \quad , \quad i = 1,2,\ldots n$$

$$X_{i+n} = (\delta_1 - \delta_i) - (\delta_1^o - \delta_i^o), \; i = 2,3,\ldots n \qquad (2.61)$$

With these variables the state model is

$$\begin{aligned} \dot{\underline{X}} &= A_N \underline{X} - B_N \underline{f}(\underline{\sigma}) \\ \underline{\sigma} &= C_N^T \underline{X} \end{aligned} \qquad (2.62)$$

where

$$A_N = \begin{bmatrix} \Lambda_{nn} & 0_{n(n-1)} \\ K^T_{n(n-1)} & 0_{(n-1)(n-1)} \end{bmatrix}; \quad B_N = \begin{bmatrix} M^{-1}K_{nm} \\ 0_{(n-1)m} \end{bmatrix} \text{ and}$$

$$C_N^T = [0_{mn} \quad J_{m(n-1)}].$$

The transfer function $W_N(s)$ in this case is given by

$$W_N(s) = K^T_{nm} \, (sI_{nn} + \Lambda_{nn})^{-1} \, M^{-1} \, K_{nm} \quad . \qquad (2.63).$$

2.7 CONCLUSIONS:

This chapter has reviewed the state models of power systems in the light of some new concepts concerning un-controllability due to system overdescription. It is because of the fact that the state models in the 2n-dimensional state space both for uniform and non-uniform damping cases, the 2m-dimensional state space for uniform

damping and (m+n) state space for non-uniform damping case, exhibit redundancy in the state variable description either explicitly or otherwise, that uncontrollability in a mathematical sense is exhibited. This is explained in this chapter on the basis of physical dependency relations among the state variables. From these the legitimate order of the state vector is obtained.

CHAPTER III

STABILITY ANALYSIS USING VECTOR LYAPUNOV FUNCTIONS

3.1 INTRODUCTION

In this chapter a new approach to the stability analysis of large scale power systems using vector Lyapunov functions is introduced. Stability oanalysis of power systems using Lyapunov's method has been carried out by several research workers [3-34] and is by now well documented in the literature. It is well known that the method obtains clearing times of faults in a one step integration of the swing equations thereby reducing the computer time considerably. Although encouraging results have generally been found [12,13,17,20], the modelling techniques had to be necessarily simple because of the difficulty in the construction of suitable Lyapunov functions with detailed models. As yet, the construction of Lyapunov function for multimachine power systems with models other than 'the constant voltage behind reactance' model is largely an unsolved problem. It therefore calls for new theoretic approaches to the problem. Two such promising approaches are

(i) Dynamic equivalencing [53-64]

(ii) Use of vector Lyapunov functions [39,51]

The first approach consists in obtaining a simplified model which is then used to construct a Lyapunov function.

In the second method the composite system is decomposed into subsystems. Scalar Lyapunov functions are constructed for these subsystems. These Lyapunov functions together with the interconnections are then used to derive conditions for the stability of the composite system. The present chapter is devoted to this method of studying the stability properties of large scale power systems.

The concept of vector Lyapunov functions was first proposed by Bellman [40] and Bailey [39] demonstrated its usefulness for studying the stability of a complex composite system using the idea of decomposition and the comparison principle [72]. The technique was restricted to a particular class of interconnections, i.e. linear interconnections only. However, it opened a new line of research in applying Lyapunov's method for large scale systems. Nonlinear interconnections were proposed by Piontkovskii et al [49] but the conditions were overly restrictive. Recent works of Siljak [42-45], Grujic and Siljak [47,48], Thompson [46], Araki and Kondo [41] and Miechel [50,51] give a good exposition of these concepts and enables one to apply the results to practical systems such as power systems. The work of Siljak [43] exhibits the nature of interconnections in a simple manner which may be useful in assessing the 'connective stability' properties of complex systems.

This new technique of stability investigation of the complex system through subsystems is a two level concept. At the outset the composite system is represented in the form of simpler subsystems and their interconnections. At the lower level, each of the subsystems is tested for its stability. A scalar Lyapunov function satisfying the necessary sign definite properties is constructed for each of them in the usual manner. On the higher hierarchial level, these scalar functions are taken as components of a vector Lyapunov function [39] defined for the composite system. Using the constraints on the interconnections between the subsystems a vector matrix differential inequality is obtained. Applying the comparison principle [72] stability of the overall system is then ascertained.

Applications of the vector Lyapunov function concept in physical systems in general and power systems in particular do not seem to have appeared in the literature so far. The power system, being a large complex system, is a natural candidate for using such a technique. This was hinted at in reference[35] .The main impediment in the successful application of the theory appears to be the difficulty in achieving an effective decomposition procedure of the complex system such that the subsystems have the desirable properties required for the construction of a vector Lyapunov function. Most of the examples cited in

the literature presuppose the existence of subsystems together with their interactions. But in a physical system an exactly reverse situation exists: a complex system is given and the first step is to obtain a suitable decomposition before applying the theory.

This chapter therefore addresses itself to the problem of proposing a decomposition technique for large scale power systems and demonstrating the application of the vector Lyapunov function approach for the stability analysis of the composite system corresponding to the post-fault state. A scalar Lyapunov function of the Lure'-Popov form is then obtained for the composite system from the vector Lyapunov function to estimate the stability region.

3.2 SOME ASPECTS OF VECTOR LYAPUNOV FUNCTIONS[42-45,47,48]:

Consider a continuous autonomous dynamical system $\mathcal{S}$ described by the vector differential equation

$$\dot{\underline{X}} = \underline{f}(\underline{X}) \qquad (3.1)$$

where $\underline{X}$ is an n-dimensional state vector and the vector function $\underline{f}(\underline{X})$ satisfies the Lipschitz conditions so that the solutions of (3.1) exist and are unique and continuous for all initial conditions $\underline{X}_o$ of the state vector $\underline{X}$. Also $\underline{f}(\underline{0}) = \underline{0}$ and the origin $\underline{X} = \underline{0}$ is the unique equilibrium point of the system (3.1).

Let the system $\mathcal{S}$ be decomposed into s dynamical subsystems $\mathcal{S}_i$ described by

$$\left.\begin{aligned} \dot{\underline{X}}_i &= \underline{f}_i(\underline{X}_i) + \sum_{j=1}^{s} e_{ij}\, \underline{g}_{ij}(\underline{Y}_j) \\ \underline{Y}_i &= \underline{h}_i(\underline{X}_i) \end{aligned}\right\} , \quad i=1,2,\ldots s \qquad (3.2)$$

where $\underline{X}_i$ is the state vector of the subsystem of order n_i,

$\underline{f}_i(\underline{X}_i)$ is a vector valued function of order n_i,

e_{ij} are interconnection numbers to be defined later,

$\underline{g}_{ij}(\underline{Y}_j)$ is a vector valued function of order n_i,

$\underline{Y}_i$ is the output of the subsystem of order m_i,

and $\underline{h}_i(\underline{X}_i)$ is a vector function of order m_i.

It is assumed that the interacting function $\underline{g}_{ij}(\underline{Y}_j)$ and the function $\underline{h}_j(\underline{X}_j)$ are such that

$$\| \underline{g}_{ij}(\underline{h}_j(\underline{X}_j)) \| \leq \eta_{ij} \; \| \underline{X}_j \| \qquad (3.3)$$

where η_{ij} are non-negative numbers and $\|\underline{X}_j\|$ denotes the Eucledean norm of $\underline{X}_j$ given by $(\underline{X}_j^T \; \underline{X}_j)^{\frac{1}{2}}$.

The state vector of the system $\mathcal{S}$ is given by

$$\underline{X} = [\underline{X}_1^T, \; \underline{X}_2^T, \; \ldots \; \underline{X}_s^T]^T. \qquad (3.4)$$

Accordingly the dimensions of $\mathcal{S}$ and $\mathcal{S}_i$, i=1,2,...s are related by

$$n = \sum_{i=1}^{n} n_i \; . \qquad (3.5)$$

The subsystem (3.2) will be defined as 'forced subsystem' with the vector functions $\underline{g}_{ij}(\underline{Y}_j)$ representing the action

of the jth subsystem on the ith subsystem. If the summation terms on the right hand side of (3.2) are equated to zero, the subsystem description reduces to

$$\begin{aligned} \dot{\underline{X}}_i &= \underline{f}_i(\underline{X}_i) \\ \underline{y}_i &= \underline{h}_i(\underline{X}_i) \ . \end{aligned} \qquad (3.6)$$

Then (3.6) defines the 'free' subsystem $\mathcal{S}_i$, i=1,2,...s. The interconnection numbers e_{ij} assume values of either zero or one depending on whether or not $\mathcal{S}_j$ acts on $\mathcal{S}_i$. Thus

$$\begin{aligned} e_{ij} &= 1 \quad \text{if } \mathcal{S}_j \text{ acts on } \mathcal{S}_i \\ e_{ij} &= 0 \quad \text{if } \mathcal{S}_j \text{ does not act on } \mathcal{S}_i . \end{aligned} \qquad (3.7)$$

Using these numbers, it is possible to exhibit the interconnections existing between the various subsystems through an 'interconnection matrix', $\mathcal{C}$, and also illustrate the structural changes that can take place. For example consider the system shown in Figure 3.1. It consists of two subsystems $\mathcal{S}_1$ and $\mathcal{S}_2$ of order n_1 and n_2 respectively. With all the switches closed, the interconnection matrix $\mathcal{C}$ is given by

$$\mathcal{C} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} ,$$

indicating that all the interactions g_{ij}, i,j = 1,2 are on. Other forms of $\mathcal{C}$ are possible such as $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ etc., depending on whether the corresponding g_{ij}'s act or not.

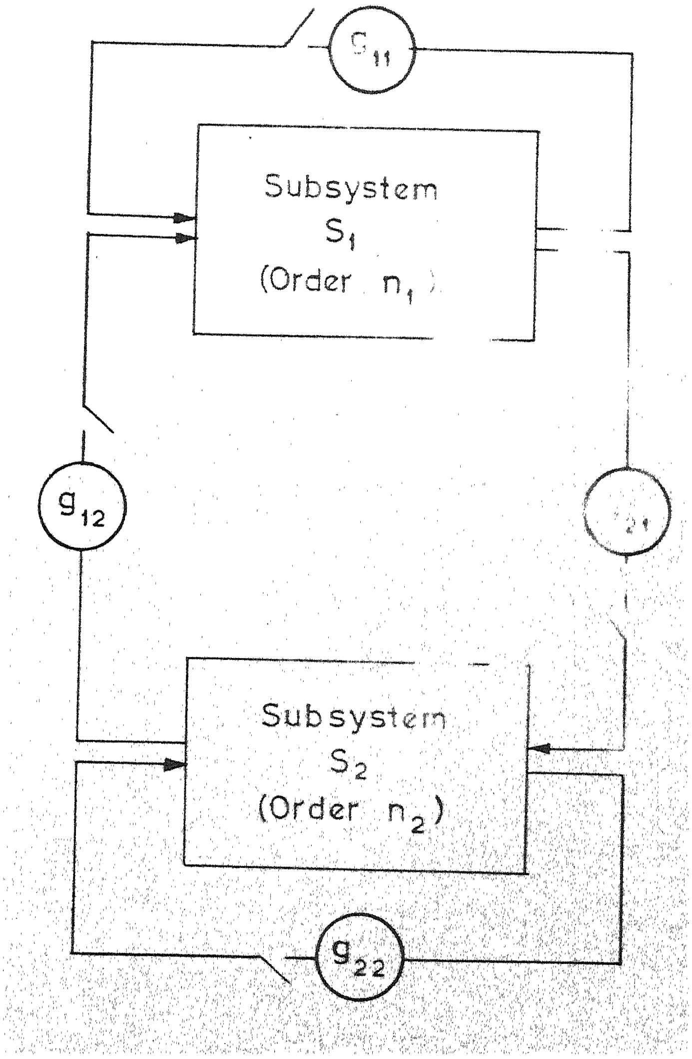

FIG. 3.1. SUBSYSTEM INTERCONNECTIONS

It will be assumed that each of the free subsystems (3.6) are asymptotically stable. For such systems, the following theorem holds [73].

THEOREM 3.1: Let the equilibrium of the system of differential equations

$$\dot{\underline{X}}_i = \underline{f}_i(\underline{X}_i) \tag{3.8}$$

be asymptotically stable. Then there exists a positive definite decrescent function $V_i(\underline{X}_i)$ with a negative definite derivative.

Proof of this theorem is contained in reference [73]. As a result of this theorem, $V_i(\underline{X}_i)$ satisfies the following conditions

$$\begin{aligned} \Phi_{i1}(\|\underline{X}_i\|) \leq V_i(\underline{X}_i) &\leq \Phi_{i2}(\|\underline{X}_i\|) \\ \dot{V}_i(\underline{X}_i) &\leq -\Phi_{i3}(\|\underline{X}_i\|) \end{aligned} \tag{3.9}$$

where $\dot{V}_i(\underline{X}_i) = (\text{Grad } V_i)^T \underline{f}_i(\underline{X}_i)$, is the total time derivative of $V_i(\underline{X}_i)$ along the solutions of (3.8) and Grad V_i is the gradient vector given by

$$(\text{Grad } V_i)^T = \left[\frac{\partial V_i}{\partial X_{i1}}, \frac{\partial V_i}{\partial X_{i2}} \cdots \frac{\partial V_i}{\partial X_{in_i}} \right].$$

The X_{ij}'s, $j=1,2...n_i$ are the elements of $\underline{X}_i$. The functions Φ_{i1}, Φ_{i2} and Φ_{i3} are comparison functions belonging to class κ. Some definitions in connection with theorem (3.1) are given in Appendix A for completeness and clarity.

In order to infer stability of the composite system it is necessary to establish conditions that the interacting functions have to satisfy. It will be therefore assumed that there exist bounded functions $\xi_{ij}(\underline{X}_j)$ such that

$$(\text{grad } V_i)^T g_{ij}(\underline{X}_j) \leq \sum_{j=1}^{s} \xi_{ij}(\underline{X}_j) \; \Phi_{j3}(\|\underline{X}_j\|) \,, \qquad i=1,2,\ldots s. \qquad (3.10)$$

Existence of these conditions crucially depends upon the nature of the Lyapunov function $V_i(\underline{X}_i)$ and the interacting functions. These in turn depend upon the way in which the system $\mathcal{S}$ is decomposed.

The properties of $V_i(\underline{X}_i)$ given by (3.9) and the inequalities governing the interconnecting functions $g_{ij}(\underline{X}_j)$ given by (3.10) are now unified by taking the total time derivative $\dot{\tilde{V}}_i(\underline{X}_i)$ of $V_i(\underline{X}_i)$ along the solutions of the 'forced' subsystem $(\mathcal{S}_i)$. This gives

$$\begin{aligned} \dot{\tilde{V}}_i(\underline{X}_i) &= \dot{V}_i(\underline{X}_i) + (\text{grad } V_i)^T g_{ij}(\underline{X}_j) \\ &\leq -\Phi_{i3}(\|\underline{X}_i\|) + \sum_{j=1}^{s} \xi_{ij}(\underline{X}_j) \, \Phi_{j3}(\|\underline{X}_j\|) \,, \end{aligned}$$

$$i = 1,2,\ldots s \,. \qquad (3.11)$$

Now define a vector Lyapunov function $\underline{V}_v$ as

$$\underline{V}_v = [V_1 \; V_2 \; \ldots \; V_s]^T \qquad (3.12)$$

and a comparison vector function $\underline{W}$ as

$$\underline{W} = [\Phi_{13}, \; \Phi_{23} \; \ldots \; \Phi_{s3}]^T \,. \qquad (3.13)$$

Using (3.12) and (3.13) in equations (3.11) the following vector matrix differential inequality is obtained.

$$\dot{\underline{V}}_v \leq R\,\underline{W} \tag{3.14}$$

where $\dot{\underline{V}}_v = [\dot{\hat{V}}_1, \ \dot{\hat{V}}_2, \ \ldots \ \dot{\hat{V}}_s]^T$. (3.15)

The elements r_{ij} of the matrix R are given by

$$r_{ij} = -\delta^*_{ij} + \hat{\alpha}_{ij} \tag{3.16}$$

where δ^*_{ij} is the kroncker delta symbol with

$$\delta^*_{ij} = 1 \quad \text{for} \quad i = j$$

and $\quad \delta^*_{ij} = 0 \quad \text{for } i \neq j.$

The $\hat{\alpha}_{ij}$'s are given by [48]

$$\hat{\alpha}_{ij} = \delta^*_{ij} \operatorname{Sup} \xi_{ij}(\underline{X}_j) + (1-\delta^*_{ij}) \max(0, \operatorname{Sup} \xi_{ij}(\underline{X}_j)). \tag{3.17}$$

Finally to conclude stability of the composite system, the matrix R of (3.14) with its elements r_{ij} given by (3.16) is tested in terms of the following theorem [42,45,48].

THEOREM 3.2: The equilibrium state $\underline{X} = \underline{0}$ of the composite system $\mathcal{S}$ is asymptotically stable in the large if the sxs matrix $R = (r_{ij})$ defined by (3.16) satisfies the conditions

$$r_{11} < 0, \quad \begin{vmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{vmatrix} > 0, \ \ldots \ (-1)^s \begin{vmatrix} r_{11} & r_{12} & \cdots & r_{1s} \\ r_{21} & r_{22} & \cdots & r_{2s} \\ \cdot & \cdot & \cdot & \cdot \\ r_{s1} & r_{s2} & \cdots & r_{ss} \end{vmatrix} > 0 \tag{3.18}$$

Proof of this theorem is contained in reference [48], and is included in Appendix B .

3.3 SOME PROPERTIES OF THE R-MATRIX:

1. Every element r_{ij} of R is related to a single interconnection between two subsystems $\mathcal{S}_i$ and $\mathcal{S}_j$.

2. The matrix R has non-negative off-diagonal elements. For such matrices the following Lemma holds[74].

LEMMA 2.1: A real sxs matrix R $(= r_{ij})$ with $r_{ij} \geq 0$ (for all $i,j = 1,2,\ldots s$, $i \neq j$) has all eigenvalues λ^*_k with negative real parts, i.e.

$$\mathrm{Re}(\lambda^*_k) < 0 \ , \ \text{for all } k = 1,2,\ldots s,$$

if and only if the inequalities (3.18) is satisfied.

According to Lemma 2.1, the matrix R satisfying (3.18), is Hurwitz.

3. The matrix R is also negative definite.

4. The negative of matrix R, i.e., -R, has all off-diagonal elements non-positive. Such matrices are called 'Metzler' matrices [48,75] and have extensively been used in economic theory [76].

3.4 SCALAR LYAPUNOV FUNCTION FROM VECTOR LYAPUNOV FUNCTION:

In this section a scalar Lyapunov function $V_s(\underline{X})$ will be constructed for the composite system using the vector Lyapunov function. This scalar function is useful in estimating the stability domain for the overall system.

Such a region exists whenever the Lyapunov functions $V_i(\underline{X}_i)$ of the subsystems do not satisfy conditions (3.9) in their entire state spaces but satisfy only in a certain region around the origins of their respective state spaces.

We choose a function $V_s(\underline{X})$ of the type

$$V_s(\underline{X}) = \underline{d}^T \underline{V}_v \tag{3.20}$$

as a candidate for the scalar Lyapunov function for the composite system where $\underline{d}$ is a constant s-vector $(d_1, d_2, .. d_s)^T$ with its elements $d_i > 0$. Since every element of $\underline{d}$ is positive and every element of $\underline{V}_v$ is positive definite, V_s is also positive definite [48]. Also, taking the time derivative of $V_s(\underline{X})$ yields

$$\dot{V}_s(\underline{X}) = \underline{d}^T \dot{\underline{V}}_v \leq \underline{d}^T R \underline{W} \tag{3.21}$$

Since R has non-negative off-diagonal elements and satisfies (3.18) R is Hurwitz. Hence for any given s-vector $\underline{b} > 0$, it is possible to find the vector $\underline{d}$ of the form shown such that

$$\underline{b}^T = -\underline{d}^T R . \tag{3.22}$$

Therefore (3.21) gives

$$\dot{V}_s(\underline{X}) < -\underline{b}^T \underline{W} \tag{3.23}$$

which shows $\dot{V}_s(\underline{X}) < 0$. Therefore (3.20) indeed constitutes a Lyapunov function for the overall system.

When the estimates on the subsystem Lyapunov function $V_i(\underline{X}_i)$, i = 1,2,...s are linear (i.e. $\Phi_{ij} = \eta_{ij} \| \underline{X}_i \|$, j = 1,2,3 ; i = 1,2,..s), it is possible to use the following theorem [48] to obtain a scalar Lyapunov function $V_s(\underline{X})$ of the form $\underline{V}_v^T H \underline{V}_v$ for the composite system.

THEOREM 3.3: The equilibrium state $\underline{X} = \underline{0}$ of the composite system, $\mathcal{S}$ is asymptotically stable in the large if there exists a symmetric positive definite sxs matrix Q such that the symmetric positive definite sxs matrix H (= h_{ij}) which is a solution of the Lyapunov matrix equation

$$R^T H + H R = -Q \tag{3.24}$$

has all its elements specified as

$$\begin{aligned} h_{ij} &\geq 0 \quad , i \neq j \\ h_{ij} &> 0 \quad , i = j . \end{aligned} \tag{3.25}$$

Proof of this theorem is contained in reference [48].

Since the estimates on V_i and $\mathring{V}_i$ are linear, it is possible to obtain a relationship of the form

$$V_i(\underline{X}_i) \geq K_i \Phi_{i3} \; , \; i = 1,2,...s \tag{3.26}$$

where K_i's are constants greater than zero. Then $\underline{W}$ can be related to $\underline{V}_v$ by the inequality

$$\underline{V}_v(\underline{X}) \geq K_i^* \underline{W} \tag{3.27}$$

where K_i^* = Min K_i, i = 1,2,...s. Now choose a vector function $\underline{Z}$

$$\underline{Z}(\underline{V}_v) = H\,\underline{V}_v \tag{3.28}$$

where H is a positive definite symmetric matrix and consider a function $V_s(\underline{X})$ of the form

$$V_s(\underline{X}) = 2 \sum_{i=1}^{s} \int_0^{V_i} (H\,\underline{V}_v)_i\, d\,V_i \tag{3.29}$$

as a Lyapunov function of the composite system. Expanding (3.29) one obtains

$$\begin{aligned} V_s(\underline{X}) = 2[&\int_0^{V_1} [h_{11}V_1(\underline{X}_1) + h_{12}V_2(\underline{X}_2) + \dots + h_{1s}\, V_s(\underline{X}_s)]dV_1 \\ + &\int_0^{V_2} [h_{21}V_1(\underline{X}_1) + h_{22}V_2(\underline{X}_2) + \dots + h_{2s}V_s(\underline{X}_s)]d\,V_2 \\ + &\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ + &\int_0^{V_s} [h_{s1}V_1(\underline{X}_1) + h_{s2}V_2(\underline{X}_2) + \dots + h_{ss}V_s(\underline{X}_s)]d\,V_s]. \end{aligned}$$

On performing the integrations,

$$\begin{aligned} V_s(\underline{X}) &= h_{11}\, V_1^2 + 2h_{12}V_1V_2 + \dots + h_{ss}V_s^2 \\ &= \underline{V}_v^T\, H\, \underline{V}_v \quad . \end{aligned} \tag{3.30}$$

Clearly $V_s(\underline{X})$ is positive definite. Taking the time derivative of (3.30)

$$\begin{aligned} \dot{V}_s(\underline{X}) &= 2\ \underline{Z}^T(\underline{V}_v)\ \dot{\underline{V}}_v \\ &= 2\ \underline{V}_v^T\ H\ \dot{\underline{V}}_v \end{aligned}$$

Using (3.14) we obtain the inequality

$$\dot{V}_s(\underline{X}) \le 2\ \underline{V}_v^T\ H\ R\ \underline{W} \tag{3.31}$$

and in view of (3.27),

$$\begin{aligned}\dot{V}_s(\underline{X}) &\le 2K_i^* \underline{W}^T H R \underline{W} \\ &\le K_i^*[\underline{W}^T R^T H \underline{W} + \underline{W}^T H R \underline{W}] \\ &\le -K_i^* \underline{W}^T Q \underline{W} . \end{aligned} \tag{3.32}$$

Since R is Hurwitz and H is positive definite symmetric, by assumption, a positive definite Q always exists which is a solution of the Lyapunov matrix equation

$$R^T H + H R = -Q . \tag{3.33}$$

Hence $\dot{V}_s(\underline{X}) < 0$. It may be noted that, the restriction on the elements h_{ij} of H have been relaxed since the estimates on $V_i(\underline{X})$ and $\dot{V}_i(\underline{X})$ are linear.

Since $V_s(\underline{X}) > 0$ and $\dot{V}_s(\underline{X}) < 0$, $V_s(\underline{X})$ is indeed a Lyapunov function for the dynamical system $\mathcal{S}$.

With this background, it is now possible to analyze the stability problem of the large scale power system.

3.5 THE POWER SYSTEM PROBLEM:

3.5.1 Decomposition:

The swing equations characterizing an n-machine power system as indicated in equations (2.9) of Chapter II are given by

$$M_i \frac{d^2\delta_i}{dt^2} + D_i \frac{d\delta_i}{dt} = (P_{mi}-E_i^2 G_{ii}) - \sum_{\substack{j=1 \\ j\neq i}}^{n} E_i E_j B_{ij} \sin(\delta_i-\delta_j), \quad i=1,2,\ldots n . \tag{3.34}$$

As shown in Chapter II, Section 2.3, these equations can be cast into a vector matrix differential equation of the form

$$\begin{aligned} \dot{\underline{X}} &= A\,\underline{X} - B\,\underline{f}(\underline{\sigma}) \\ \underline{\sigma} &= C^T\,\underline{X} \end{aligned} \qquad (3.35)$$

where the state vector $\underline{X}$ and the vector $\underline{\sigma}$, and the matrices A,B and C are all of proper order depending on the state space in which the system is represented. Various forms of these models have been explained in Chapter II. A key to the application of the vector Lyapunov function is the ability to decompose the composite system into subsystems. The system models in the minimal state space i.e., (2n-1) for non-uniform damping and (2n-2) for uniform damping are not amenable to a decomposition where the interactions are functions of the state variables of the interacting subsystems as required in equation (3.2). Hence decomposition in the minimal state space was not possible. However the system descriptions discussed in Chapter II in the n(n-1) and (n+m) state spaces for uniform and non-uniform damping cases respectively lend themselves to an effective decomposition procedure. Therefore with these models, the decomposition procedure will be illustrated for a three machine system initially and will be later generalised to an n-machine case.

Case (i) 3-machine system with uniform damping ($\lambda_i = \lambda$, i=1,2,3):

In this case the state model as derived in Section 2.5.2 of Chapter II is of the form

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \cdot \\ \cdot \\ \cdot \\ \dot{X}_6 \end{bmatrix} = \begin{bmatrix} -\lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & -\lambda & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ \cdot \\ X_6 \end{bmatrix}$$

$$- \begin{bmatrix} \frac{1}{M_1} + \frac{1}{M_2} & \frac{1}{M_1} & -\frac{1}{M_2} \\ \frac{1}{M_1} & \frac{1}{M_1} + \frac{1}{M_3} & \frac{1}{M_3} \\ -\frac{1}{M_2} & \frac{1}{M_3} & \frac{1}{M_2} + \frac{1}{M_3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} f_1(\sigma_1) \\ f_2(\sigma_2) \\ f_3(\sigma_3) \end{bmatrix}$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_6 \end{bmatrix} \qquad (3.36)$$

with the state variables defined as

$$X_1 = \omega_1 - \omega_2 \quad ; \quad X_4 = (\delta_1 - \delta_2) - (\delta_1^o - \delta_2^o)$$
$$X_2 = \omega_1 - \omega_3 \quad ; \quad X_5 = (\delta_1 - \delta_3) - (\delta_1^o - \delta_3^o) \tag{3.37}$$
$$X_3 = \omega_2 - \omega_3 \quad ; \quad X_6 = (\delta_2 - \delta_3) - (\delta_2^o - \delta_3^o) \ .$$

The $f_i(\sigma_i)$'s, i=1,2,3 are the same as those defined in equation (2.38) of Chapter II. Decomposition into the 3 subsystems is achieved by choosing the state variables from equation (3.37) as follows:

SUBSYSTEM 1: Choose

$$X_1 = \omega_1 - \omega_2$$
$$X_4 = (\delta_1 - \delta_2) - (\delta_1^o - \delta_2^o) \tag{3.38}$$

as the state variables for this subsystem. Extracting equations corresponding to these states from (3.36), we get

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} -\lambda & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_4 \end{bmatrix} - \begin{bmatrix} \frac{1}{M_1} + \frac{1}{M_2} \\ 0 \end{bmatrix} f_i(\sigma_1) - \begin{bmatrix} \frac{1}{M_1} \\ 0 \end{bmatrix} f_2(\sigma_2)$$
$$+ \begin{bmatrix} \frac{1}{M_2} \\ 0 \end{bmatrix} f_3(\sigma_3)$$
$$\sigma_1 = [0 \quad 1] \begin{bmatrix} X_1 \\ X_4 \end{bmatrix} \ . \tag{3.39}$$

SUBSYSTEM 2: Choosing

$$\begin{aligned} X_2 &= \omega_1 - \omega_3 \\ X_5 &= (\delta_1 - \delta_3) - (\delta_1^o - \delta_3^o) \end{aligned} \qquad (3.40)$$

as the state variables of this subsystem, and extraction of the state equations corresponding to these variables from (3.36) results in the description of the forced subsystem 2 as

$$\begin{bmatrix} \dot{X}_2 \\ \dot{X}_5 \end{bmatrix} = \begin{bmatrix} -\lambda & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} X_2 \\ X_5 \end{bmatrix} - \begin{bmatrix} \frac{1}{M_1} + \frac{1}{M_3} \\ 0 \end{bmatrix} f_2(\sigma_2) - \begin{bmatrix} \frac{1}{M_1} \\ 0 \end{bmatrix} f_1(\sigma_1)$$

$$- \begin{bmatrix} \frac{1}{M_3} \\ 0 \end{bmatrix} f_3(\sigma_3)$$

$$\sigma_2 = [0 \quad 1] \begin{bmatrix} X_2 \\ X_5 \end{bmatrix} \qquad (3.41)$$

SUBSYSTEM 3: Adopting the same procedure as for the first two subsystems the state space description of this subsystem with the state variables

$$\begin{aligned} X_3 &= \omega_2 - \omega_3 \\ X_6 &= (\delta_2 - \delta_3) - (\delta_2^o - \delta_3^o) \end{aligned} \qquad (3.42)$$

will be of the form

$$\begin{bmatrix} \dot{X}_3 \\ \dot{X}_6 \end{bmatrix} = \begin{bmatrix} -\lambda & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} X_3 \\ X_6 \end{bmatrix} - \begin{bmatrix} \frac{1}{M_2} + \frac{1}{M_3} \\ 0 \end{bmatrix} f_3(\sigma_3) + \begin{bmatrix} \frac{1}{M_2} \\ 0 \end{bmatrix} f_1(\sigma_1)$$

$$- \begin{bmatrix} \frac{1}{M_3} \\ 0 \end{bmatrix} f_2(\sigma_2) \tag{3.43}$$

$$\sigma_3 = [0 \quad 1] \begin{bmatrix} X_3 \\ X_6 \end{bmatrix} .$$

The system (3.36) is thus decomposed into 3 subsystems described by (3.39), (3.41) and (3.43). These equations together constitute the composite system analogous to (3.2) and thus represent the dynamics of the 'forced' subsystems 1, 2 and 3 respectively. The last two terms on the right hand side of the equations (3.39), (3.41) and (3.43) are identified as the interactions among the subsystems. The description of the 'free' subsystems will then be as follows:

SUBSYSTEM 1:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} -\lambda & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_4 \end{bmatrix} - \begin{bmatrix} \frac{1}{M_1} + \frac{1}{M_2} \\ 0 \end{bmatrix} f_1(\sigma_1)$$

$$\sigma_1 = [0 \quad 1] \begin{bmatrix} X_1 \\ X_4 \end{bmatrix} \tag{3.44}$$

SUBSYSTEM 2:

$$\begin{bmatrix} \dot{X}_2 \\ \dot{X}_5 \end{bmatrix} = \begin{bmatrix} -\lambda & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} X_2 \\ X_5 \end{bmatrix} - \begin{bmatrix} \frac{1}{M_1} + \frac{1}{M_3} \\ 0 \end{bmatrix} f_2(\sigma_2)$$

$$\sigma_2 = [0 \quad 1] \begin{bmatrix} X_2 \\ X_5 \end{bmatrix} \tag{3.45}$$

and

SUBSYSTEM 3:

$$\begin{bmatrix} \dot{X}_3 \\ \dot{X}_6 \end{bmatrix} = \begin{bmatrix} -\lambda & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} X_3 \\ X_6 \end{bmatrix} - \begin{bmatrix} \frac{1}{M_2} + \frac{1}{M_3} \\ 0 \end{bmatrix} f_3(\sigma_3)$$

$$\sigma_3 = [0 \quad 1] \begin{bmatrix} X_3 \\ X_6 \end{bmatrix} \tag{3.46}$$

It is evident from the above equations that the free subsystems represent the dynamics of a hypothetical 2-machine system cast in the Lure'-Popov form. Thus subsystem 1 consists of machines 1 and 2, subsystem 2 includes machines 1 and 3 and subsystem 3 comprises of machines 2 and 3. It may be noted that the decomposition is a mathematical one and not a physical one as in Kron's 'tearing technique'. The interactions among the subsystems take place through the second and third terms on the right hand sides of equations (3.39), (3.41) and (3.43). These are nonlinear

due to nonlinear nature of $f_i(\sigma_i)$ occurring in them. The interconnection numbers e_{ij} exhibiting these interactions among the 3 subsystems are given by the following interconnection matrix $\mathscr{C}$.

$$\mathscr{C} = \begin{array}{c} \\ (1) \\ (2) \\ (3) \end{array} \begin{array}{c} \begin{array}{ccc} (1) & (2) & (3) \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{array} \tag{3.47}$$

This matrix $\mathscr{C}$ is invariant in structure. A block diagram representation of the interconnected system $\mathcal{S}$, with the subsystems and their interactions explicitly displayed is shown in Figure (3.2).

Case (ii) 3-machine with non-uniform damping ($\lambda_i \neq \lambda_j$, $i,j = 1,2,3$):

The state model of a 3-machine system obtained in Section 2.5.3 of Chapter II in the (m+n) dimensional state space with the state variables defined by

$$\begin{aligned} X_1 &= \omega_1 \;; & X_4 &= (\delta_1 - \delta_2) - (\delta_1^o - \delta_2^o) \\ X_2 &= \omega_2 \;; & X_5 &= (\delta_1 - \delta_3) - (\delta_1^o - \delta_3^o) \\ X_3 &= \omega_3 \;; & X_6 &= (\delta_2 - \delta_3) - (\delta_2^o - \delta_3^o) \end{aligned} \tag{3.48}$$

is of the form

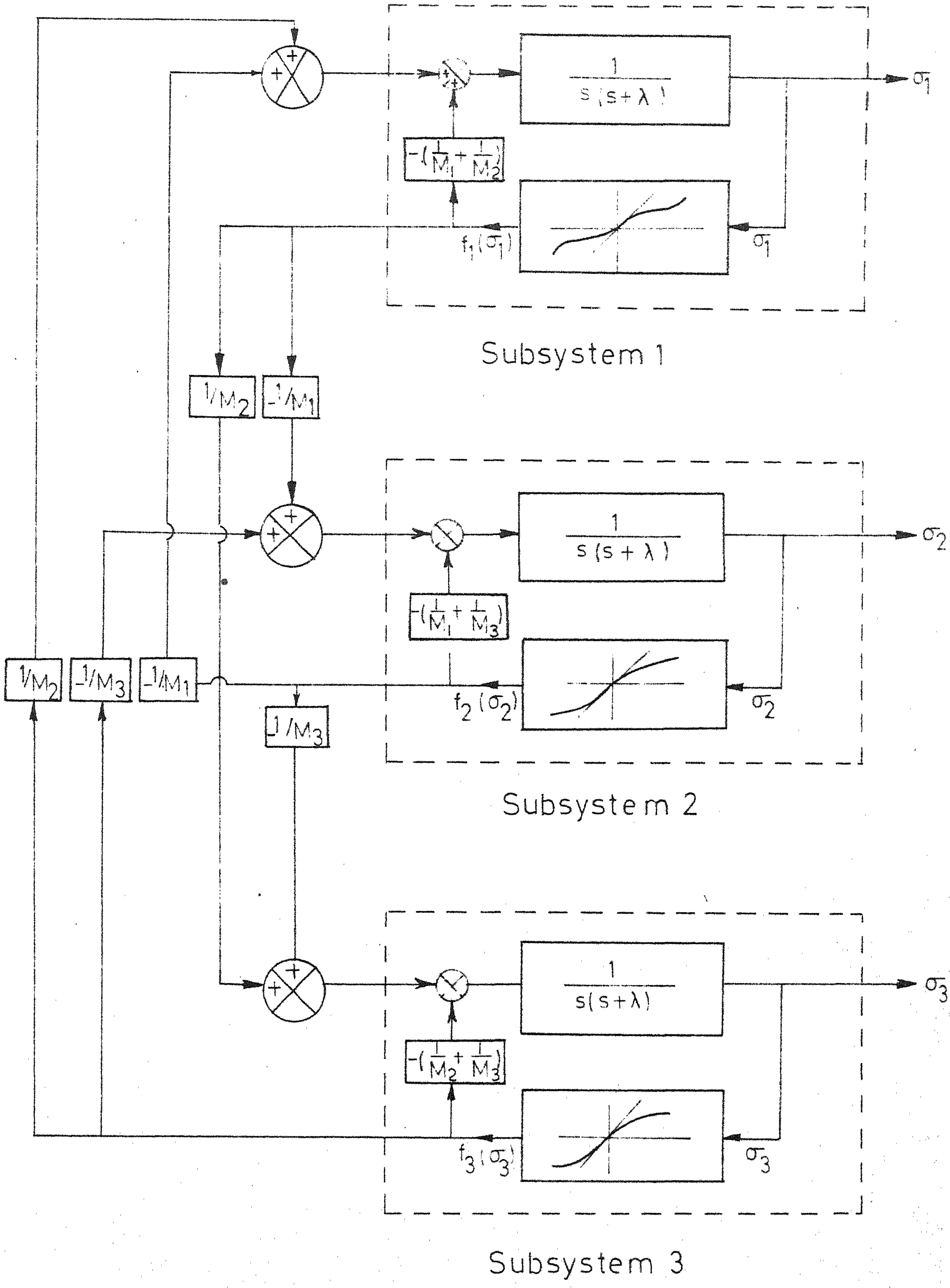

FIG.3.2. SUBSYSTEM INTERCONNECTIONS IN A 3MACHINE POWER SYSTEM (UNIFORM DAMPING)

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \cdot \\ \cdot \\ \dot{X}_6 \end{bmatrix} = \begin{bmatrix} -\lambda_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\lambda_3 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ \cdot \\ \cdot \\ X_6 \end{bmatrix}$$

$$- \begin{bmatrix} \frac{1}{M_1} & \frac{1}{M_1} & 0 \\ -\frac{1}{M_2} & 0 & \frac{1}{M_2} \\ 0 & -\frac{1}{M_3} & -\frac{1}{M_3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} f_1(\sigma_1) \\ f_2(\sigma_2) \\ f_3(\sigma_3) \end{bmatrix} \tag{3.49}$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ X_6 \end{bmatrix} .$$

Direct decomposition of (3.49) into subsystems described by equations of the type (3.2) where the interacting functions $\underline{g}_{ij}(\underline{Y}_{ij})$ are functions of the state of the jth subsystem only is not possible. This difficulty is overcome by augmenting the first three state equations

$$\dot{X}_1 = -\lambda_1 X_1 - \frac{1}{M_1} f_1(\sigma_1) - \frac{1}{M_1} f_2(\sigma_2)$$

$$\dot{X}_2 = -\lambda_2 X_2 + \frac{1}{M_2} f_1(\sigma_1) - \frac{1}{M_2} f_3(\sigma_3)$$

and $$\dot{X}_3 = -\lambda_3 X_3 + \frac{1}{M_3} f_2(\sigma_2) + \frac{1}{M_3} f_3(\sigma_3) \;.$$

Effectively, this implies that the state description of the composite system is in the $(m+3m)$ dimensional state space. This does not, however, affect the results and is only an artifice to achieve the desired decomposition.

Decomposition into three subsystems is now straightforward and is carried out in the following way:

SUBSYSTEM 1: The state variables selected for this subsystem are

$$\begin{aligned} X_1 &= \omega_1 \\ X_2 &= \omega_2 \\ X_4 &= (\delta_1 - \delta_2) - (\delta_1^o - \delta_2^o) \end{aligned} \tag{3.50}$$

Extracting the corresponding equations from (3.49), subsystem 1 is described by

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} -\lambda_1 & 0 & 0 \\ 0 & -\lambda_2 & 0 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_4 \end{bmatrix} - \begin{bmatrix} \frac{1}{M_1} \\ \frac{1}{M_2} \\ 0 \end{bmatrix} f_1(\sigma_1) - \begin{bmatrix} \frac{1}{M_1} \\ 0 \\ 0 \end{bmatrix} f_2(\sigma_2)$$

$$\begin{bmatrix} 0 \\ \frac{1}{M_2} \\ 0 \end{bmatrix} f_3(\sigma_3)$$

$$\sigma = [0 \quad 0 \quad 1] \begin{bmatrix} X_1 \\ X_2 \\ X_4 \end{bmatrix} \tag{3.51}$$

SUBSYSTEM 2: Here the state variables are

$$\begin{aligned} X_1 &= \omega_1 \\ X_3 &= \omega_3 \\ X_5 &= (\delta_1 - \delta_3) - (\delta_1^0 - \delta_3^0) \end{aligned} \tag{3.52}$$

The state equations are then described by

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_3 \\ \dot{X}_5 \end{bmatrix} = \begin{bmatrix} -\lambda_1 & 0 & 0 \\ 0 & -\lambda_3 & 0 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_3 \\ X_5 \end{bmatrix} - \begin{bmatrix} \frac{1}{M_1} \\ -\frac{1}{M_3} \\ 0 \end{bmatrix} f_2(\sigma_2) - \begin{bmatrix} \frac{1}{M_1} \\ 0 \\ 0 \end{bmatrix} f_1(\sigma_1)$$

$$- \begin{bmatrix} 0 \\ -\frac{1}{M_3} \\ 0 \end{bmatrix} f_3(\sigma_3) \tag{3.53}$$

$$\sigma_2 = [0 \quad 0 \quad 1] \begin{bmatrix} X_1 \\ X_3 \\ X_5 \end{bmatrix} .$$

SUBSYSTEM 3: By a procedure analogous to the previous cases this subsystem is described by

$$\begin{bmatrix} \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_6 \end{bmatrix} = \begin{bmatrix} -\lambda_2 & 0 & 0 \\ 0 & -\lambda_3 & 0 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} X_2 \\ X_3 \\ X_6 \end{bmatrix} - \begin{bmatrix} \frac{1}{M_2} \\ -\frac{1}{M_3} \\ 0 \end{bmatrix} f_3(\sigma_3) - \begin{bmatrix} -\frac{1}{M_2} \\ 0 \\ 0 \end{bmatrix} f_1(\sigma_1)$$

$$- \begin{bmatrix} 0 \\ -\frac{1}{M_3} \\ 0 \end{bmatrix} f_2(\sigma_2)$$

$$\sigma_3 = [0 \quad 0 \quad 1] \begin{bmatrix} X_2 \\ X_3 \\ X_6 \end{bmatrix} \tag{3.54}$$

with the state variables given by

$$\begin{aligned} X_2 &= \omega_2 \\ X_3 &= \omega_3 \\ X_6 &= (\delta_2 - \delta_3) - (\delta_2^o - \delta_3^o) \end{aligned} \tag{3.55}$$

Thus the 3-machine system is decomposed into 3 subsystems described by (3.51), (3.53)and (3.54). Each of these represents a 'forced' subsystem analogous to the system (3.2). Again with the last two terms on the right hand sides of these equations identified as 'interactions', the 'free' subsystems are described by:

SUBSYSTEM 1:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} -\lambda_1 & 0 & 0 \\ 0 & -\lambda_2 & 0 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_4 \end{bmatrix} - \begin{bmatrix} \frac{1}{M_1} \\ -\frac{1}{M_2} \\ 0 \end{bmatrix} f_1(\sigma_1) \; ; \; \sigma_1 = [0 \; 0 \; 1] \begin{bmatrix} X_1 \\ X_2 \\ X_4 \end{bmatrix} \tag{3.56}$$

SUBSYSTEM 2:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_3 \\ \dot{X}_5 \end{bmatrix} = \begin{bmatrix} -\lambda_1 & 0 & 0 \\ 0 & -\lambda_3 & 0 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_3 \\ X_5 \end{bmatrix} - \begin{bmatrix} \frac{1}{M_1} \\ -\frac{1}{M_3} \\ 0 \end{bmatrix} f_2(\sigma_2) \; ; \; \sigma_2 = [0 \; 0 \; 1] \begin{bmatrix} X_1 \\ X_3 \\ X_5 \end{bmatrix} \tag{3.57}$$

and

SUBSYSTEM 3:

$$\begin{bmatrix} \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_6 \end{bmatrix} = \begin{bmatrix} -\lambda_2 & 0 & 0 \\ 0 & -\lambda_3 & 0 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} X_2 \\ X_3 \\ X_6 \end{bmatrix} - \begin{bmatrix} \frac{1}{M_2} \\ -\frac{1}{M_3} \\ 0 \end{bmatrix} f_3(\sigma_3); \quad \sigma_3 = [0 \ 0 \ 1] \begin{bmatrix} X_2 \\ X_3 \\ X_6 \end{bmatrix} \tag{3.58}$$

Each of these free subsystem models can be identified with that of an equivalent 2 machine system with non-uniform damping. The first comprises machines 1 and 2, the second 1 and 3 and the third 2 and 3. A block diagram representation of the composite system exhibiting the subsystems and their interactions is given in Figure 3.3. Finally, the interactions represented by the last two terms on the right hand sides of (3.51), (3.53) and (3.54) are exhibited by the interconnection matrix $\mathcal{G}$ of equation (3.47).

The procedure of decompositin described will now be generalized to an n-machine system for both the uniform and the non-uniform damping cases.

Case (iii) n-machine system with uniform damping:

State model in the 2m-dimensional state space for the n-machine case was derived in Section 2.5.2 of Chapter II. Decomposition of this system into 'm' subsystems is carried out in a manner outlined for the 3-machine case. Then a typical ith subsystem involving machine 'k' and 'j'

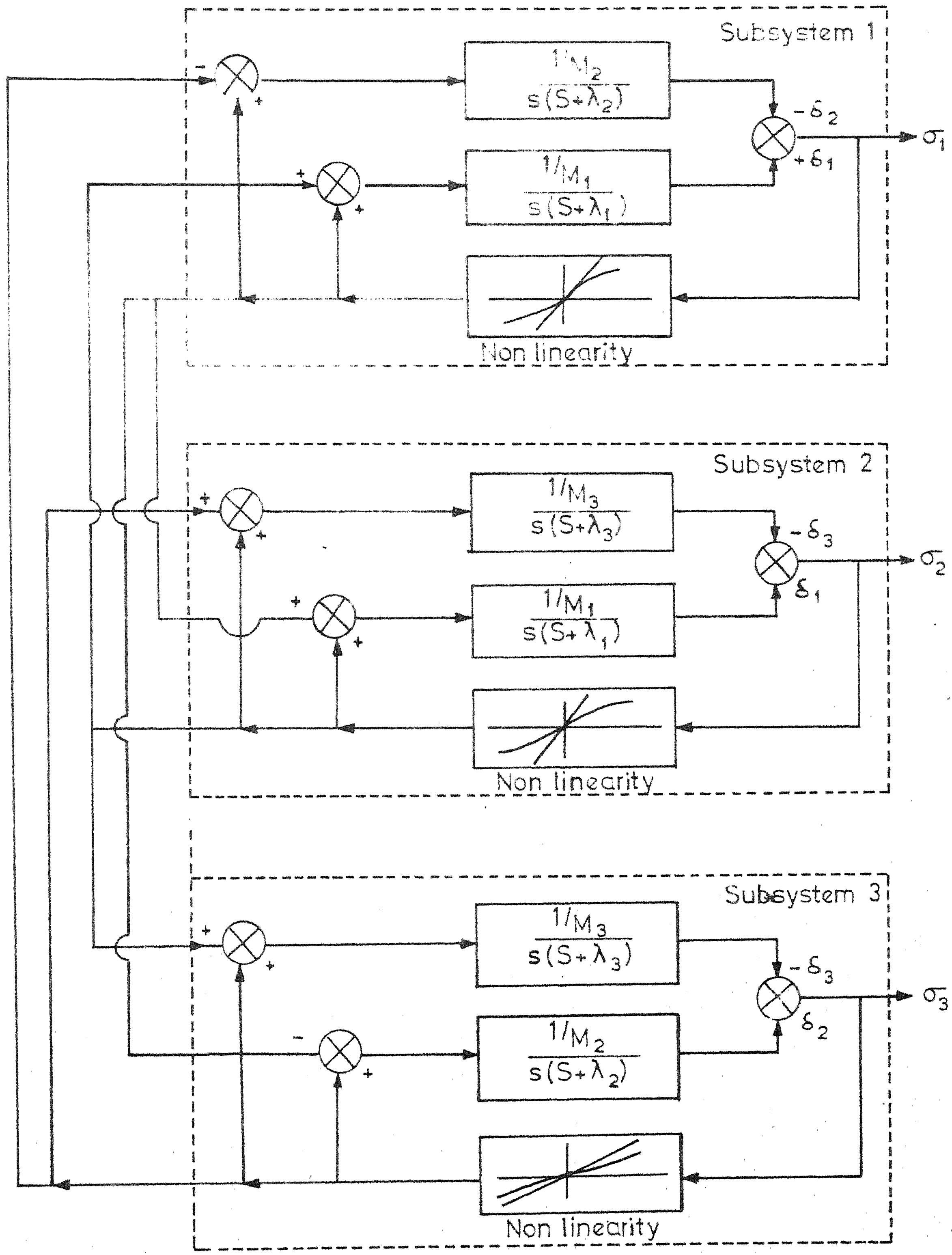

FIG. 3.3. SUBSYSTEM INTERCONNECTIONS IN A 3 MACHINE POWER SYSTEM (NON-UNIFORM DAMPING)

is of the form

$$\dot{\underline{X}}_i = A_i \underline{X}_i - \underline{B}_i f_i(\sigma_i) + \sum_{\substack{q=1 \\ q\neq i}}^{m} e_{iq} \underline{B}_q \underline{f}_q(\sigma_q) \qquad i=1,2,\ldots m \tag{3.59}$$

$$\sigma_i = \underline{C}_i^T \underline{X}_i$$

where $\underline{X}_i$ is the subsystem state vector. The matrix A_i and vectors $\underline{C}_i$, $\underline{B}_i$ and $\underline{B}_q$ are obtained as

$$A_i = \begin{bmatrix} -\lambda & 0 \\ 1 & 0 \end{bmatrix} ; \quad C_i = [0 \quad 1]^T \quad ; \underline{B}_i = \begin{bmatrix} \frac{1}{M_k} + \frac{1}{M_j} \\ 0 \end{bmatrix}$$

$$\text{and} \quad \underline{B}_q = \pm \begin{bmatrix} \frac{1}{M_k} \\ 0 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} \frac{1}{M_j} \\ 0 \end{bmatrix} . \tag{3.60}$$

As before the numbers e_{iq} constitute the elements of the interconnection matrix $\mathscr{C}$. This matrix is of the form

$$\mathscr{C}_{mm} = (K_{nm}^T K_{nm} - 2I_{mm}) , \tag{3.61}$$

where I is an identity of order m.

The free subsystem $\mathcal{S}_i$ is finally described by

$$\dot{\underline{X}}_i = A_i \underline{X}_i - \underline{B}_i f_i(\sigma_i) \; ; \; \sigma_i = \underline{C}_i^T \underline{X}_i \tag{3.62}$$

and represents the dynamics of a 2-machine system.

Case (iv) n-machine system with non-uniform damping:

The state model of the n-machine case with non-uniform damping in the (m+n) dimensional state space developed in

Section 2.5.3 will be used here to obtain a suitable decomposition into 'm' subsystems. Adopting a procedure identical to the 3-machine case, a typical ith subsystem involving the kth and jth machines ($k < j$) will be as follows:

$$\dot{\underline{X}}_i = \tilde{A}_i \underline{X}_i - \tilde{\underline{B}}_i f_i(\sigma_i) + \sum_{\substack{q=1 \\ q \neq i}}^{m} e_{iq} \tilde{\underline{B}}_q f_q(\sigma_q)$$

$$\sigma_i = \tilde{\underline{C}}_i^T \underline{X}_i \qquad i = 1,2,\ldots m \tag{3.63}$$

where $\underline{X}_i$ is the subsystem state vector of order 3. The matrix $\tilde{A}_i$ and vectors $\tilde{\underline{C}}_i$, $\tilde{\underline{B}}_i$ and $\tilde{\underline{B}}_q$ are:

$$\tilde{A}_i = \begin{bmatrix} -\lambda_k & 0 & 0 \\ 0 & -\lambda_j & 0 \\ 1 & -1 & 0 \end{bmatrix}; \quad \tilde{\underline{B}}_i = \begin{bmatrix} \frac{1}{M_k} \\ -\frac{1}{M_j} \\ 0 \end{bmatrix}; \quad \tilde{\underline{C}}_i = [\,0 \quad 1\,]^T$$

$$\text{and } \tilde{\underline{B}}_q = \pm \begin{bmatrix} \frac{1}{M_k} \\ 0 \\ 0 \end{bmatrix} \text{ or } \pm \begin{bmatrix} 0 \\ \frac{1}{M_j} \\ 0 \end{bmatrix}. \tag{3.64}$$

The interconnection structure among the subsystems is exhibited by the interconnection matrix $\mathcal{C}$ given by (3.61). Lastly the free subsystem is of the form

$$\dot{\underline{X}}_i = \tilde{A}_i \underline{X}_i - \tilde{\underline{B}}_i f_i(\sigma_i)$$

$$\sigma_i = \tilde{\underline{C}}_i^T \underline{X}_i \tag{3.65}$$

which again represents the dynamics of a two machine system with non-uniform damping. This completes the decomposition procedure.

3.5.2 Construction of Lyapunov function for subsystems:

For the free subsystems described in Section 3.5.1 scalar Lyapunov functions will now be constructed. Each of these subsystems is in the Lure'-Popov form with a single nonlinearity. The pairs (A_i, $\underline{B}_i$) and (A_i, $\underline{C}_i$) are easily verified to be completely controllable and completely observable. Hence the triplet (A_i, $\underline{B}_i$, $\underline{C}_i$) constitutes a minimal realization [69] for the hypothetical 2-machine subsystem. The transfer function of the linear part of these subsystems are given in equations (2.32) and (2.28) of Chapter II both for uniform and non-uniform damping cases. Techniques for generating Lyapunov functions for such systems are well known [7-10,14,15]. Here we use the **Moore-Anderson** theorem for constructing the scalar Lyapunov function for each of the subsystems. This theorem is stated without proof in Appendix C. Following the method of reference [10], a Lyapunov function $V_i(\underline{X}_i)$ of the type

$$V_i(\underline{X}_i) = \underline{X}_i^T P_i \underline{X}_i + 2\beta_i \int_0^{\sigma_i} f_i(s_i) ds_i \qquad (3.66)$$

is constructed in the respective state spaces of the subsystems $\mathcal{S}_i$, i=1,2,...s.

For the uniform damping case, the matrix P_i is given by

$$P_i = \frac{M_k M_j}{M_k + M_j} \begin{bmatrix} 2/\lambda & 1 \\ 1 & \lambda \end{bmatrix} \qquad (3.67)$$

with $\beta_i = \frac{2}{\lambda}$. Similarly for the non-uniform damping case the matrix P_i takes the form

$$P_i = \begin{bmatrix} \beta_i M_k & 0 & \frac{D_k D_j}{\lambda_k (D_k + D_j)} \\ 0 & \beta_i M_j & -\frac{D_k D_j}{\lambda_j (D_k + D_j)} \\ \frac{D_k D_j}{\lambda_k (D_k + D_j)} & -\frac{D_k D_j}{\lambda_j (D_k + D_j)} & \frac{D_k D_j}{D_k + D_j} \end{bmatrix} \quad (3.68)$$

and $\beta_i = (\frac{1}{\lambda_k} + \frac{1}{\lambda_j})$.

The time derivative $\dot{V}(\underline{X})$ of this Lyapunov function is given by [10]

$$\dot{V}_i(\underline{X}_i) = -\underline{X}_i^T L_i L_i^T \underline{X}_i - 2f_i(\sigma_i)\,\sigma_i \quad (3.69)$$

and negative definite for all σ_i lying in the open interval $(-\pi-2\theta_i, \pi-2\theta_i)$ and the positive semidefinite matrix $L_i L_i^T$ is obtained from the solution of the Lyapunov matrix equation

$$A^T P + P A = - L L^T . \quad (3.70)$$

This implies that the subsystem is asymptotically stable in the region of interest. It is precisely this property that has resulted in a successful application of the vector Lyapunov function to the stability analysis of the composite power system.

3.5.3 Stability Regions:

The utility of Lyapunov functions in power system stability analysis lies in the computation of

their stability domains. In this section methods of determining these regions are discussed.

The nonlinearities occurring in power system models are known to satisfy the Popov's sector conditions

$$0 \leq \sigma_i f_i(\sigma_i) \leq \infty \;, \; i = 1,2,\ldots m$$
$$f_i(\sigma_i) = 0 \quad \text{for } \sigma_i = 0 \tag{3.71}$$

for only those σ_i that lie in the closed interval

$$-\ell_{1i} \leq \sigma_i \leq \ell_{2i}$$
$$\text{where } \ell_{1i} = \pi+2(\delta_k^o - \delta_j^o) \tag{3.72}$$
$$\ell_{2i} = \pi-2(\delta_k^o - \delta_j^o) \;.$$

Therefore a region of stability exists around the origin of the state space of the subsystem $\mathcal{S}_i$. Hence for the composite system too there exists a region of stability. This region can be estimated by a knowledge of the stability regions of the individual subsystems [77]. Methods are available to compute these regions [67,68]. Alternatively, the scalar Lyapunov function constructed in Section 3.4 can be used to estimate the stability domain for the composite system directly. While the former technique needs further investigation, the latter procedure is adopted here to compute the stability regions of the composite system. A recently proposed method [31] is

applied to estimate the nearest unstable equilibrium point for this purpose.

We shall now illustrate the complete procedure with reference to a 3-machine system.

3.6 APPLICATION TO A 3-MACHINE SYSTEM:

3.6.1 Decomposition and application of vector Lyapunov method:

The example chosen to illustrate the application of the vector Lyapunov functions is the 3-machine system (Figure 3.4) of reference [5]. The system data is given in Table 3.1 and the transfer admittances between the internal nodes of the machines are as follows:

$$y_{12} = 0 + j2 \text{ p.u.}$$

$$y_{13} = 0 + j1 \text{ p.u.} \tag{3.73}$$

and $y_{23} = 0 + j3$ p.u.

A disturbance is simulated by changing y_{23} from 3 p.u. to 1 p.u. and assigning arbitrary velocities to machines.

Table 3.1 - SYSTEM DATA

Machine i	Voltage E_i (p.u.)	Mech.Power Input (P_{mi}) (p.u.)	Inertia Constant M_i (p.u.)
1	1.0	1.5	0.02
2	1.0	0.0	0.002
3	1.0	-1.5	0.03

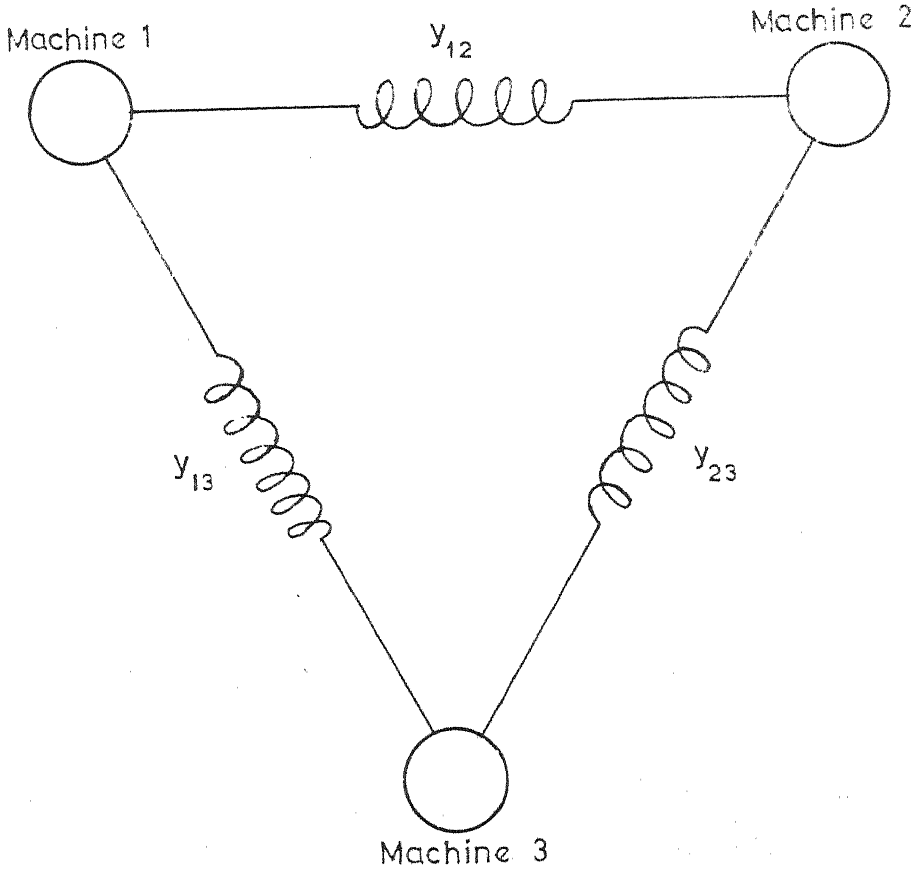

FIG. 3.4. THE THREE MACHINE SYSTEM

For this composite system the post-fault stable equilibrium point is given by [5]

$$\begin{aligned} \delta_1^o - \delta_2^o &= 18.78^o \\ \delta_1^o - \delta_3^o &= 58.86^o \qquad (3.74) \\ \text{and} \quad \delta_2^o - \delta_3^o &= 40.08^o. \end{aligned}$$

With uniform damping and λ=1 assumed, the state representation of the system according to equation (3.36) is

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \cdot \\ \cdot \\ \cdot \\ \dot{X}_6 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ \cdot \\ X_6 \end{bmatrix}$$

$$- \begin{bmatrix} 550 & 50.00 & -500.00 \\ 50 & 83.33 & 33.33 \\ -500 & 33.33 & 533.33 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} f_1(\sigma_1) \\ f_2(\sigma_2) \\ f_3(\sigma_3) \end{bmatrix} \qquad (3.75)$$

$$\underline{\sigma} = [0_{3x3} \mid I_{3x3}] \, \underline{X}$$

Decomposition yields the following 3 forced systems:

SUBSYSTEM 1:

$$\dot{\underline{X}}_1 = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix} \underline{X}_1 - \begin{bmatrix} 550 \\ 0 \end{bmatrix} f_1(\sigma_1) - \begin{bmatrix} 50 \\ 0 \end{bmatrix} f_2(\sigma_2) + \begin{bmatrix} 500 \\ 0 \end{bmatrix} f_3(\sigma_3) \qquad (3.76)$$

$$\sigma_1 = [0 \quad 1]\underline{X}_1$$

SUBSYSTEM 2:

$$\dot{\underline{X}}_2 = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix} \underline{X}_2 - \begin{bmatrix} 83.33 \\ 0 \end{bmatrix} f_2(\sigma_2) - \begin{bmatrix} 50 \\ 0 \end{bmatrix} f_1(\sigma_1) - \begin{bmatrix} 33.33 \\ 0 \end{bmatrix} f_3(\sigma_3) \qquad (3.77)$$

$$\sigma_2 = [0 \; 1] \; \underline{X}_2$$

and

SUBSYSTEM 3:

$$\dot{\underline{X}}_3 = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix} \underline{X}_3 - \begin{bmatrix} 533.3 \\ 0 \end{bmatrix} f_3(\sigma_3) - \begin{bmatrix} 500 \\ 0 \end{bmatrix} f_1(\sigma_1) - \begin{bmatrix} 33.33 \\ 0 \end{bmatrix} f_2(\sigma_2) \qquad (3.78)$$

$$\sigma_3 = [0 \quad 1] \; \underline{X}_3 \; .$$

The 'free' subsystems are given by

SUBSYSTEM 1:

$$\dot{\underline{X}}_1 = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix} \underline{X}_1 - \begin{bmatrix} 550 \\ 0 \end{bmatrix} f_1(\sigma_1) \; ; \quad \sigma_1 = [0 \; 1] \; \underline{X}_1 \qquad (3.79)$$

SUBSYSTEM 2:

$$\dot{\underline{X}}_2 = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix} \underline{X}_2 - \begin{bmatrix} 83.33 \\ 0 \end{bmatrix} f_2(\sigma_2) \; ; \quad \sigma_2 = [0 \; 1] \; \underline{X}_2 \qquad (3.80)$$

and

SUBSYSTEM 3:

$$\dot{\underline{X}}_3 = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix} \underline{X}_3 - \begin{bmatrix} 533.3 \\ 0 \end{bmatrix} f_3(\sigma_3) \; ; \quad \sigma_3 = [0 \; 1] \, \underline{X}_3 \quad (3.81)$$

In equations (3.75) - (3.81), the nonlinearities $f_i(\sigma_i)$, $i = 1,2,3$ are

$$f_1(\sigma_1) = 2.0 \, [\sin(\sigma_1+(\delta_1^o-\delta_2^o)) - \sin(\delta_1^o-\delta_2^o)]$$
$$f_2(\sigma_2) = 1.0 \, [\sin(\sigma_2+(\delta_1^o-\delta_3^o)) - \sin(\delta_1^o-\delta_3^o)] \quad (3.82)$$
$$f_3(\sigma_3) = 1.0 \, [\sin(\sigma_3+(\delta_2^o-\delta_3^o)) - \sin(\delta_2^o-\delta_3^o)] \; .$$

These nonlinearities satisfy the Popov's sector condition for σ_i, $i = 1,2,3$ lying in the closed interval

$$-3.7974 \le \sigma_1 \le 2.4858$$
$$-5.1970 \le \sigma_2 \le 1.0862 \quad (3.83)$$
$$-4.5412 \le \sigma_3 \le 1.7420 \quad .$$

Also it is easily verified that the origins of the free subsystems (3.79) - (3.81) are asymptotically stable. Hence for each of these subsystems Lyapunov function of the type

$$V_i(\underline{X}_i) = \underline{X}_i^T \, P_i \, \underline{X}_i + 4 \int_0^{\sigma_i} f_i(s_i) ds_i, \; 1=1,2,3 \quad (3.84)$$

are constructed. The matrices P_i for each of these functions are

$$P_1 = \begin{bmatrix} 0.00364 & 0.00182 \\ 0.00182 & 0.00182 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 0.024 & 0.012 \\ 0.012 & 0.012 \end{bmatrix} \tag{3.85}$$

$$\text{and } P_3 = \begin{bmatrix} 0.00374 & 0.00187 \\ 0.00187 & 0.00187 \end{bmatrix} .$$

We choose a function $v_i(\underline{X}_i) = V_i^{\frac{1}{2}}(\underline{X}_i)$, $i = 1,2,3$ and obtain the following linear estimates on $v_i(\underline{X}_i)$ [42,78]

$$0.0263 \quad \| \underline{X}_1 \| \leq \quad v_1(\underline{X}_1) \leq 2.0 \| \underline{X}_1 \|$$

$$0.068 \quad \| \underline{X}_2 \| \leq \quad v_2(\underline{X}_2) \leq 1.415 \| \underline{X}_2 \| \tag{3.86}$$

$$\text{and} \quad 0.0265 \quad \| \underline{X}_3 \| \leq v_3(\underline{X}_3) \leq 1.415 \| \underline{X}_3 \| .$$

The derivatives $\dot{v}_i(\underline{X}_i)$ of the scalar functions $v_i(\underline{X}_i)$, $i = 1,2,3$ along the solutions of the free subsystems are

$$\dot{v}_1(\underline{X}_1) = - \frac{1}{2v_1} (0.00364 \ X_1^2 + 2f_1(\sigma_1)\sigma_1) = -\Phi_{13}(\| \underline{X}_1 \|)$$

$$\dot{v}_2(\underline{X}_2) = - \frac{1}{2v_2}(0.024 \ X_2^2 + 2f_2(\sigma_2) \ \sigma_2) = -\Phi_{23}(\| \underline{X}_2 \|)$$

$$\dot{v}_3(\underline{X}_3) = - \frac{1}{2v_3}(0.00374 \ X_3^2 + 2f_3(\sigma_3)\sigma_3) = -\Phi_{33}(\| \underline{X}_3 \|) . \tag{3.87}$$

The Φ_{i3}'s, $i = 1,2,3$ will be identified as comparison functions.

NOTE: The derivatives $\dot{v}_i(\underline{X}_i)$ of (3.87) are negative definite for $\sigma_i, i=1,2,3$ lying in the open intervals (-3.7974, 2.54858), (-5.1970, 1.0862) and (-4.5412, 1.7420) respectively.

Now the derivatives $\dot{\tilde{v}}_i(\underline{X}_i)$, $i = 1,2,3$ of $v_i(\underline{X}_i)$ along the solutions of the forced subsystems are computed. This gives

$$\dot{\tilde{v}}_1(\underline{X}_1) = \dot{v}_1(\underline{X}_1) + \frac{0.00182}{v_1}(2X_1+X_4)\,[-50f_2(\sigma_2) + 500f_3(\sigma_3)]$$

$$= \dot{v}_1(\underline{X}_1) - \frac{0.091}{v_1}(2X_1+X_4)\,f_2(\sigma_2) + \frac{0.91}{v_1}(2X_1+X_4)f_3(\sigma_3)$$

$$\leq -\Phi_{13}(\|\underline{X}_1\|) + r_{12}\,\Phi_{23}(\|\underline{X}_2\|) + r_{13}\,\Phi_{33}(\|\underline{X}_3\|)$$

Similarly

$$\dot{\tilde{v}}_2(\underline{X}_2) \leq r_{21}\,\Phi_{13}(\|\underline{X}_1\|) - \Phi_{23}(\|\underline{X}_2\|) + r_{23}\,\Phi_{33}(\|\underline{X}_3\|)$$

and

$$\dot{\tilde{v}}_3(\underline{X}_3) \leq r_{31}\,\Phi_{13}(\|\underline{X}_1\|) + r_{32}\,\Phi_{23}(\|\underline{X}_2\|) - \Phi_{33}(\|\underline{X}_3\|) \tag{3.88}$$

where the r_{ij}'s are constants ≥ 0.

Define the vector Lyapunov function

$$\underline{v}_v = [v_1,\ v_2,\ v_3]^T \tag{3.89}$$

and the comparison vector function $\underline{W} = [\Phi_{13}, \Phi_{23}, \Phi_{33}]^T$.
Then according to equation (3.14)

$\dot{v}_v \leq R\,\underline{W}$ where

$$R = \begin{bmatrix} -1 & r_{12} & r_{13} \\ r_{21} & -1 & r_{23} \\ r_{31} & r_{32} & -1 \end{bmatrix}. \tag{3.90}$$

Computation of r_{ij}'s in the above equation is a critical part in this procedure. Majorisation procedures as used in Grujic and Siljak [48] gave rise to extremely conservative results even to the extent of indicating the system to be unstable although it is known to be steady state stable. It was threfore necessary to consider the physical aspects of the problem in arriving at sharper estimates on the r_{ij}'s. As is done in Gless [5], different initial velocities were assigned to the machines with the pre-fault rotor angles as the initial conditions. For low velocities the system was found to be stable with the assumption that the velocities decrease monotonically. For larger initial velocities the system was, however, unstable. We thus compute the r_{ij}'s by evaluating the right hand sides of (3.88) at the initial point. Table 3.2 summarizes the effect of the initial values of the state variables of the individual subsystems on the stability of the composite system. The state of the system when different initial conditions are assigned to state variables, is given in Column 5 and is compared with the state obtained in Table I of reference [5]. This comparison exhibits a tendency towards instability by this method. Thus the system stability domain is decreased with the present technique. Further numerical experimentation is called for to increase this region. One of the reasons for such a conservative estimate of the domain seems to

Table 3.2: Effect of Initial Conditions on Stability.

S.No.	Value of X_1	Value of X_2	Value of X_3	System state
1	0.03	0.09	0.06	Stable
2	0.00	0.10	0.10	Stable
3	0.025	0.075	0.05	Unstable
4	0.001	0.0705	0.0685	Stable
5	0.02	0.08	0.06	Stable
6	0.01	0.07	0.06	Stable
7	0.01	-0.05	-0.04	Unstable
8	0.04	0.01	-0.03	Unstable
9	0.06	0.05	-0.01	Unstable
10	0.01	0.09	0.08	Stable
11	0.100	0.00	-0.10	Unstable
12	0.03	-0.02	-0.05	Unstable

$$V_s(\underline{X}) = \sum_{i=1}^{3} V_i(\underline{X}_i)$$
$$\text{and} \quad \dot{V}_s(\underline{X}) \leq -K_i^* \, \underline{W}^T \, Q \, \underline{W} \tag{3.93}$$

where K_i^* is a constant greater than zero. The matrix Q as a solution of the Lyapunov matrix equation

$$R^T H + H R = - Q$$

is given by

$$Q = \begin{bmatrix} 1.00 & -0.256 & -0.932 \\ -0.256 & 1.00 & -0.112 \\ -0.932 & -0.112 & 1.00 \end{bmatrix} \tag{3.94}$$

and is positive definite and symmetric. Hence $V_s(\underline{X})$ is a Lyapunov function for the composite system. This function can be used to compute the stability region for the 3-machine system which is done in the next section.

3.6.3 Comparison with other Lyapunov functions and stability regions:

The Lyapunov function $V_s(\underline{X})$ in (3.92) **will** now be compared with the Lyapunov function constructed in the minimal state space. For a comparison to be valid both the functions have to be in the same state space [24]. To do so the Lyapunov function $V_s(\underline{X})$ will be recast in the minimal state space utilizing the linear dependency among the state variables in $\underline{X}$. Considering the state variables X_1, X_2, X_4 and X_5 to be the linearly

independent components, the function $V_s(\underline{X}_m)$ in the minimal state space would be

$$V_s(\underline{X}_m) = \underline{X}_m^T \, \underline{\tilde{P}}_m \, \underline{X}_m + \sum_{i=1}^{3} \int_0^{\sigma_i} f_i(s_i)\,ds_i \qquad (3.95)$$

where $\underline{X}_m = [\omega_1-\omega_2,\ \omega_1-\omega_3, ((\delta_1-\delta_2)-(\delta_1^o-\delta_2^o)), ((\delta_1-\delta_3)-(\delta_1^o-\delta_3^o))]^T$

and

$$\tilde{P}_m = \begin{bmatrix} 0.001845 & -0.000935 & 0.0009225 & -0.0004675 \\ -0.000935 & 0.006935 & -0.0004675 & -0.003475 \\ 0.0009225 & -0.0004675 & 0.0009225 & -0.0004675 \\ -0.0004675 & 0.003475 & -0.0004675 & 0.003475 \end{bmatrix} \qquad (3.96)$$

Direct computation using the procedure of reference [10], yields a Lyapunov function $V_m(\underline{X}_m)$ of the form

$$V_m(\underline{X}_m) = \underline{X}_m^T \, \underline{P}_m^* \, \underline{X}_m + \sum_{i=1}^{3} \int_0^{\sigma_i} f_i(s_i)\,ds_i \qquad (3.97)$$

with

$$P_m^* = \begin{bmatrix} 0.000962 & -0.000579 & 0.000321 & -0.000193 \\ -0.000579 & 0.00635 & -0.000193 & 0.00217 \\ 0.000321 & -0.000193 & 0.000321 & -0.000193 \\ -0.000193 & 0.00217 & -0.000194 & 0.00217 \end{bmatrix} \qquad (3.98)$$

and the $f_i(\sigma_i)$'s are the same as in (3.91). The structure of P_m^* is given in Appendix C.

Stability regions will now be obtained for the system using V_s and V_m by the technique of reference [31]. The estimated unstable equilibrium point $\underline{X}_m^u$ nearest to the

stable one according to this method is

$$\underline{X}_m^u = [0 \quad 0 \quad 0 \quad 1.0862]^T \tag{3.99}$$

The values of the functions $V_s(\underline{X}_m)$ and $V_m(\underline{X}_m)$ computed at this point define the stability regions. Thus

$$V_s(\underline{X}_m) \leq 0.391 \tag{3.100}$$

is a stability domain using (3.91) and

$$V_m(\underline{X}_m) \leq 0.388 \tag{3.101}$$

is the stability regions using (3.97). It cannot, however, be concluded that (3.100) yields a better result since the Lyapunov functions differ in their quadratic part. From the entries in the matrices $\tilde{P}_m$ and P_m^* it can be observed that the trajectories reach the boundaries faster using the Lyapunov function $V_s(\underline{X}_m)$. It is therefore conjectured that vector Lyapunov functions method yields conservative results than the existing methods. But this disadvantage is outweighed by the other advantages of the technique.

3.7 CONCLUSIONS:

This chapter has demonstrated the applicability of the vector Lyapunov function to the stability analysis of large scale power systems. The key to a successful application of the method lies in an effective decomposition of the power system. In this chapter a decomposition

procedure is developed both for uniform and the non-uniform damping cases. This enables one to construct Lure'-type Lyapunov functions for the subsystems. Using these functions a scalar Lyapunov function for the overall system is obtained via a vector Lyapunov function. This scalar function is compared with the existing ones. Although it yields conservative results, the possibility of including more system details makes the new approach an attractive tool for a detailed stability analysis of the large scale power system.

CHAPTER IV

TRANSIENT STABILITY REGIONS FOR MULTIMACHINE POWER SYSTEMS

4.1 INTRODUCTION:

One of the major impediments in the successful application of Lyapunov's method to practical systems is the computation of the transient stability region around the post-fault stable equilibrium point. The estimation of this region so far has been carried out by computing the unstable equilibrium point $\underline{X}^u$ closest to the post-fault stable one and evaluating the Lyapunov function $V(\underline{X})$ at this point. This determination is based on the following geometrical reasoning. The surfaces $V(\underline{X}) = \varepsilon$ where ε is a positive constant surrounding the origin $\underline{X} = \underline{0}$ are closed. As the value of ε increases these closed surfaces expand such that $V(\underline{X}) = \varepsilon_1$ is contained in $V(\underline{X}) = \varepsilon_2$ for $\varepsilon_2 > \varepsilon_1$. However, when the surface $V(\underline{X})$ = constant passes through an equilibrium point these surfaces are no longer closed. At this point

$$\dot{V}(\underline{X}) = [\text{Grad } V(\underline{X})]^T \dot{\underline{X}} = 0 \tag{4.1}$$

since $\dot{\underline{X}}$ is equal to zero at an equilibrium point. The region Ω bounded by the closed surface

$$V(\underline{X}) = V(\underline{X}^u) \tag{4.2}$$

where $\underline{X}^u$ is the unstable equilibrium point closest to the origin, i.e. the stable equilibrium point, is the region of stability. The determination of the equilibrium point $\underline{X}^u$ of the post-fault system involves the solution of

$$\underline{F}_2(\underline{X}) = 0 \tag{4.3}$$

where $\underline{F}_2(\underline{X})$ is the set of equations on the right hand side of equation (1.3). These equations are nonlinear and have (2^n-2) solutions. Of these $(2^{n-1}-1)$ are unstable solutions and one is the post-fault stable solution. While the determination of stable equilibrium is not difficult, the computation of the closest unstable equilibrium point $\underline{X}^u$ is a formidable task, and involves considerable amount of computer time. Various minimization procedure such as the steepest descent [6], Davidon-Fletcher-Powell [12], the steepest ascent [27] and Newton-Raphson [28] have been employed for the determination of these unstable equilibrium points. Some reduction in computer time was achieved by Luders [17] by approximating these points. Willems and Willems [16] defined a 2m-dimensional boundary based on the sector violation of the nonlinearities and obtained a minimum of $V(\underline{X})$ on the surface. A similar technique was proposed by Zaslavskaya etal [29] and Ananda Mohan [22]. In all

these cases the problem involves the use of some sort of a minimization technique for obtaining a minimum of $V(\underline{X})$ on the boundary. Bergen etal [34] and Di Caprio etal [30] use series expansions and obtain a somewhat conservative bound on $V(\underline{X})$. Some physical energy-type considerations were adopted by Williams etal [20]. In spite of these techniques, the computation of the region of stability has been one of the discouraging factors in the practical application of the Lyapunov theory. Recent work of Prabhakara and El-Abiad [31] may be considered as a powerful contribution in this area and enables one to apply Lyapunov methods to practical systems. The method yields approximate values of the region of stability within reasonable error bounds. Using the analogy of a single machine connected to an infinite bus, the method approximates all the unstable equilibrium points. In this chapter a theoretical basis to this technique is provided. The method utilizes the results obtained in connection with the determination of stability regions for multinonlinear systems as a generalization of the work due to Walker and Mc'Clamroch [68] and Weissenberger [67]. An alternate method that gives improved stability regions based on a conjecture due to Murthy [24] is also described for practical implementation.

4.2 REGIONS OF ATTRACTION FOR MULTINONLINEAR SYSTEMS:

4.2.1 Problem formulation:

The problem of determining the regions of stability for systems cast in the Lure'-Popov form consisting of a single nonlinearity was analyzed by Walker and Mc'Clamroch [68] and Weissenberger [67] using the sector violation information of the nonlinearity. In this section the method will be extended to a multinonlinear system.

Consider a system $\mathcal{S}$ described in an n-dimensional state space and of the form

$$\dot{\underline{X}} = A\,\underline{X} - B\,\underline{f}(\underline{\sigma}) \qquad (4.4a)$$

$$\underline{\sigma} = C^T\,\underline{X} \qquad (4.4b)$$

where A, B and C are matrices of order nxn, nxm and nxm respectively. $\underline{f}(\underline{\sigma})$ is a vector valued function of order 'm' similar to equation (2.38). (It may be noted that $m \neq n(n-1)/2$ as in the power system problem). The ith component of $\underline{f}(\underline{\sigma})$ is a function of the ith component of the m-vector $\underline{\sigma}$. $\underline{X}$ is an n-dimensional state vector.

It is assumed that (A, B, C) constitutes a minimal realization of the transfer function matrix W(s) of the system (4.4) where

$$W(s) = C^T(sI - A)^{-1} B\ . \qquad (4.5)$$

Further, let there exist constant matrices $\alpha = \mathrm{diag}(\alpha_i)$ and $Q = \mathrm{diag}(q_i)$, $\alpha_i \geq 0$, $q_i \geq 0$, $\alpha_i + q_i > 0$, such that

the function

$$Z(s) = (\alpha + Qs)\ W(s) + \alpha G^{-1} \tag{4.6}$$

is positive real [81] guranteeing the existence of a Lure'-type Lyapunov function $V(\underline{X})$ of the type

$$V(\underline{X}) = \underline{X}^T P\ \underline{X} + \sum_{i=1}^{m} 2q_i \int_0^{\sigma_i} f_i(s_i)ds_i. \tag{4.7}$$

valid in the sector

$$\left.\begin{array}{ll} 0 \le \sigma_i f_i(\sigma_i) \le g_i\sigma_i^2\ , & \sigma_i \ne 0 \\ \text{and} \quad f_i(\sigma_i) = 0 \quad , & \sigma_i = 0 \end{array}\right\},\ i=1,2,\ldots m \tag{4.8}$$

In (4.6) the matrix $G = \text{diag}(g_i)$, $i=1,2,\ldots m$. The nonlinearities $f_i(\sigma_i)$ are assumed to satisfy the Popov sector (4.8)for only those σ_i that lie in the closed interval

$$-\ell_i \le \sigma_i \le k_i\ , \quad i=1,2,\ldots m \tag{4.9}$$

as shown in Figure 4.1, k_i and ℓ_i are constants greater than zero. Letting

$2q_i = \beta_i$, $V(\underline{X})$ can be written as

$$V(\underline{X}) = \underline{X}^T P\ \underline{X} + \sum_{i=1}^{m} \beta_i \int_0^{\sigma_i} f_i(s_i)ds_i\ . \tag{4.10}$$

Since the nonlinearities $f_i(\sigma_i)$, $i = 1,2,\ldots m$ violate the Popov's sector condition (4.8) outside the interval defined in (4.9), asymptotic stability of (4.4) in the large cannot be concluded. Therefore there exists

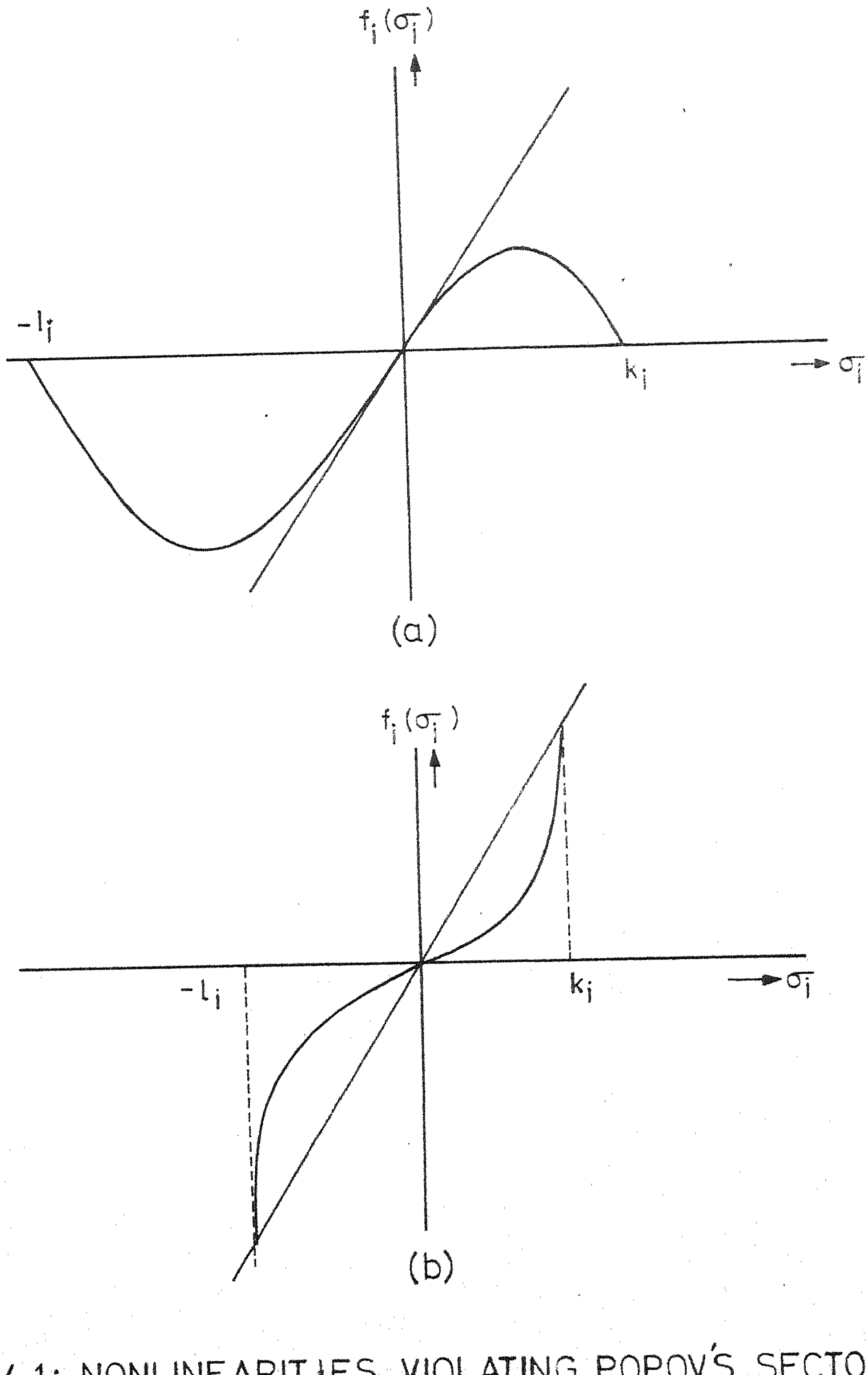

FIG. 4.1: NONLINEARITIES VIOLATING POPOV'S SECTOR

a region of stability Ω around the origin of the state space. An estimate of this domain of attraction defined by the following inequality is to be obtained,

i.e., $$V(\underline{X}) \leq \varepsilon \tag{4.11}$$

where ε is a positive constant to be determined. Clearly on the hypersurface defined by

$$V(\underline{X}) = \varepsilon \tag{4.12}$$

and its interior, the Lyapunov function and its time derivative $\dot{V}(\underline{X})$ satisfy the necessary sign definite properties namely,

$$\begin{aligned} V(\underline{X}) &> 0 \\ \dot{V}(\underline{X}) &\leq 0 \end{aligned} \tag{4.13}$$

together with grad $V(\underline{X}) \neq 0$

where $\text{grad } V(\underline{X}) = [\partial / \partial X_1, \partial / \partial X_2 \ldots \partial / \partial X_n]^T V(\underline{X})$

4.2.2 Determination of the region of attraction:

The procedure to obtain an estimate of the region of attraction using the sector violation information of the nonlinearities will now be discussed.

Let the matrix C be partitioned in the form

$$C = [\underline{C}_1, \underline{C}_2, \ldots \underline{C}_m] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & \cdots & C_{1m} \\ C_{21} & C_{22} & C_{23} & \cdots & C_{2m} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ C_{n1} & C_{n2} & C_{n3} & \cdots & C_{nm} \end{bmatrix} \tag{4.14}$$

Then equations (4.4b) and (4.14) yield

$$\sigma_i = \underline{C}_i^T \underline{X}, \quad i=1,2,\ldots m. \tag{4.15}$$

Now define 2m hyperplanes in the following way:

$$\begin{aligned} \underline{C}_i^T \underline{X} &= k_i \quad, \quad i = 1,2,\ldots m \\ \underline{C}_j^T \underline{X} &= -\ell_j \quad, \quad j = 1,2,\ldots m \end{aligned} \tag{4.16}$$

in the n-dimensional state space. Let $\mathbb{B}$ denote the boundary of the closed region $\{\underline{X}: -\ell_i \leq \underline{C}_i^T \underline{X} \leq k_i,$ $i = 1,2,\ldots m\}$. For example, in a single machine connected to an infinite bus the boundary $\mathbb{B}$ in the 2-dimensional state space is a set of parallel lines. On $\mathbb{B}$ and its interior the V-function and its time derivative satisfy the necessary sign definite properties. Hence the region contained by $\mathbb{B}$ is in the exact stability domain. A search for a minimum of $V(\underline{X})$ on this hypersurface, therefore, gives an estimate of the region of attraction. We thus find the largest of the V-surfaces that still lies inside the surface $\mathbb{B}$.

In order to establish this region consider a typical ith hyperplane of (4.16) given by

$$\sigma_i = \underline{C}_i^T \underline{X} = k_i \tag{4.17}$$

In order that (4.17) is tangential to the surface $V(\underline{X})=\varepsilon$ we use the following conditions:

$$\frac{\partial}{\partial X_j} V(\underline{X}) = \tau_i \frac{\partial}{\partial X_j}(\underline{C}_i^T \underline{X} - k_i), \; j = 1,2,..n \tag{4.18}$$

where τ_i is a nonzero constant. Equations (4.18) imply that the slope of $V(\underline{X})$ in the jth direction is a linear multiple of the slope of the hyperplane in the jth direction. Expanding (4.18), the following set of equations are obtained.

$$2P_{11}X_1 + 2P_{12}X_2 + \dots + 2P_{1i}X_i + \dots + 2P_{1n}X_n$$
$$+ \beta_1 C_{11} f_1(\sigma_1) + \beta_2 C_{12} f_2(\sigma_2) + \dots + \beta_m C_{1m} f_m(\sigma_m)$$
$$= \tau_i \; C_{1i}$$

$$2P_{21}X_1 + 2P_{22}X_2 + \dots + 2P_{2i}X_i + \dots + 2P_{2n}X_n$$
$$+ \beta_1 C_{21} f_1(\sigma_1) + \beta_2 C_{22} f_2(\sigma_2) + \dots + \beta_m C_{2m} f_m(\sigma_m)$$
$$= \tau_i \; C_{2i}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$2P_{n1}X_1 + 2P_{n2}X_2 + \dots + 2P_{ni}X_i + \dots + 2P_{nn}X_n$$
$$+ \beta_1 C_{n1} f_1(\sigma_1) + \beta_2 C_{n2} f_2(\sigma_2) + \dots + \beta_m C_{nm} f_m(\sigma_m)$$
$$= \tau_i \; C_{ni} \tag{4.19}$$

Equations (4.19) can be compactly written in the form

$$2P \; \underline{X} + C \; \beta \; \underline{f}(\underline{\sigma}) = C \; \underline{\tau} \tag{4.20}$$

where $\beta = \text{diag } (\beta_i)$, $i = 1,2,\dots m$ and $\underline{\tau}$ is an m-vector given by $[0, 0, \dots \tau_i \dots 0]^T$. (4.19) or (4.20) together with (4.17) constitute a set of (n+1) nonlinear equations in (n+1) unknowns $\underline{X}$ and τ_i and call for a numerical technique for a solution. To these equations the (m-1)

equations obtained from (4.4b) by deleting the ith equation, i.e., $\underline{\sigma}_i = \underline{C}_i^T \underline{X}$ are appended. These (n+m) equations can now be cast in a vector matrix equation of the type

$$\begin{bmatrix} 2P & -C \\ C^T & 0 \end{bmatrix} \begin{bmatrix} \underline{X} \\ \underline{\tau} \end{bmatrix} = \begin{bmatrix} -C\beta \underline{f}(\underline{\sigma}) \\ \underline{\sigma} \end{bmatrix} \qquad (4.21)$$

The unknowns in (4.21) are $\underline{X}$, τ_i and all components of $\underline{\sigma}$ except σ_i which is equal to k_i. Let it be assumed that the solution of (4.21) yields a state vector $\hat{\underline{X}}$ and a corresponding m-vector $\hat{\underline{\sigma}}$. Substitution of these into equation (4.10) yields the minimum value of $V(\underline{X})$ on the hyperplane given by (4.17). An expression for this minimum of $V(\underline{X})$ explicitly in $\underline{\sigma}$ can be obtained in the following way:

Transposing equation (4.20) and post-multiplying by $\underline{X}$ and rearranging, we obtain

$$2\hat{\underline{X}}^T P \hat{\underline{X}} - \underline{\tau}^T C^T \hat{\underline{X}} = -[\beta \underline{f}(\hat{\underline{\sigma}})]^T C^T \hat{\underline{X}} .$$

Using (4.4b) this equation simplifies to

$$2\hat{\underline{X}}^T P \hat{\underline{X}} - \underline{\tau}^T \hat{\underline{\sigma}} = -[\beta \underline{f}(\hat{\underline{\sigma}})]^T \hat{\underline{\sigma}}$$

Therefore

$$\hat{\underline{X}}^T P \hat{\underline{X}} = \tfrac{1}{2}[\underline{\tau}^T \hat{\underline{\sigma}} - [\beta \underline{f}(\hat{\underline{\sigma}})]^T \hat{\underline{\sigma}}]$$

Hence $$V(\hat{\underline{X}}) = \tfrac{1}{2}[\underline{\tau}^T \hat{\underline{\sigma}} - [\beta \underline{f}(\hat{\underline{\sigma}})]^T \hat{\underline{\sigma}}] + \sum_{i=1}^{m} \beta_i \int_0^{\hat{\sigma}_i} f_i(s_i)ds_i \qquad (4.22)$$

To eliminate $\underline{\tau}$ consider equation (4.20) which gives with $\underline{X} = \hat{\underline{X}}$ and $\underline{\sigma} = \hat{\underline{\sigma}}$,

$$\hat{\underline{X}} = \frac{P^{-1}}{2}[C\underline{\tau} - C\ \beta\ \underline{f}(\hat{\underline{\sigma}})]$$

Premultiplying by C^T gives

$$C^T\ \hat{\underline{X}} = \tfrac{1}{2}C^T\ P^{-1}[C\underline{\tau} - C\ \beta\ \underline{f}(\hat{\underline{\sigma}})]$$

or
$$\hat{\underline{\sigma}} = \tfrac{1}{2}C^T\ P^{-1}[C\ \underline{\tau} - C\ \beta\ \underline{f}(\hat{\underline{\sigma}})]$$

$$= \tfrac{1}{2}C^T\ P^{-1}\ C\underline{\tau} - \tfrac{1}{2}C^T\ P^{-1}\ C[\beta\ \underline{f}(\hat{\underline{\sigma}})]$$

$$\therefore \qquad \underline{\tau} = 2[C^T\ P^{-1}\ C]^{-1}\ \underline{\sigma} + \beta\ \underline{f}(\hat{\underline{\sigma}}) \tag{4.23}$$

Substituting for $\underline{\tau}$ in (4.22) $V(\hat{\underline{X}})$ is given by

$$V(\hat{\underline{X}}) = \hat{\underline{\sigma}}^T[C^T\ P^{-1}\ C]^{-1}\ \hat{\underline{\sigma}} + \sum_{i=1}^{m} \beta_i \int_0^{\hat{\sigma}_i} f_i(s_i)ds_i \tag{4.24}$$

Thus $V(\hat{\underline{X}})$ is explicit in $\hat{\underline{\sigma}}$ and is useful in computing the values of $V(\hat{\underline{X}})$ quickly from a knowledge of the sector violation information. Since there are 2m hyperplanes of the type (4.17) as shown in (4.16) the minimum of all the 2m values of $V(\hat{\underline{X}})$ obtained using the above procedure yields ε which defines the region of attraction for the system (4.4) according to (4.11).

4.2.3 Special cases:

(i) The single nonlinearity case:

The case of a system with a single nonlinearity (m=1) can now be considered as a particular case of (4.4). Choosing

$$\hat{\sigma} = \mathrm{Min}(k_1, \ell_1)$$

equation (4.21) yields

$$\begin{bmatrix} 2P & -C \\ C^T & 0 \end{bmatrix} \begin{bmatrix} \hat{\underline{X}} \\ \bar{\tau} \end{bmatrix} = \begin{bmatrix} -C\ \beta_1\ f(\hat{\sigma}) \\ \hat{\sigma} \end{bmatrix} \tag{4.25}$$

where $\bar{\tau}$ and β_1 are nonzero scalars. These equations constitute a set of (n+1) equations in $\underline{X}$ and $\bar{\tau}$ and can be explicitly solved to obtain

$$\varepsilon = V_{min}(\hat{\underline{X}}) = \frac{\hat{\sigma}^2}{(C^T P^{-1} C)} + \int_0^{\hat{\sigma}} \beta_1\ f(s)\ ds \tag{4.26}$$

which is exactly identical to the results obtained by Walker and Mc'Clamroch [68] and Weissenberger [67].

(ii) Integrals of nonlinearities neglected (β=0):

Since $\beta_i = 0$ ($i = 1,2,\ldots m$), $V(\underline{X})$ becomes

$$V(\underline{X}) = \underline{X}^T\ P\ \underline{X} \tag{4.27}$$

Equations (4.21) then becomes

$$\begin{bmatrix} 2P & -C \\ C^T & 0 \end{bmatrix} \begin{bmatrix} \underline{X} \\ \underline{\tau} \end{bmatrix} = \begin{bmatrix} \underline{0} \\ \underline{\sigma} \end{bmatrix} \tag{4.28}$$

(4.28) constitutes a set of (n+m) linear algebraic equations in $\underline{X}$, τ_i and (m-1) components of $\underline{\sigma}$ that are to be determined. Explicit expression for $V(\hat{\underline{X}})$ in terms of the specified value k_i for σ_i is possible in the following way:

For all $\beta_i = 0$, $i = 1,2,\ldots m$, equations (4.19) yield

$$2P\,\underline{X} - \underline{C}_i\,\tau_i = 0 \tag{4.29}$$

With (4.17), (4.29) can be put in the form

$$\begin{bmatrix} 2P & -\underline{C}_i \\ \underline{C}_i^T & 0 \end{bmatrix} \begin{bmatrix} \underline{X} \\ \tau_i \end{bmatrix} = \begin{bmatrix} 0 \\ k_i \end{bmatrix} . \tag{4.30}$$

Solution of (4.30) gives

$$\tau_i = 2\,\frac{\det[P]}{\det[H]}\,k_i \tag{4.31}$$

where $H = \begin{bmatrix} 2P & -\underline{C}_i \\ \underline{C}_i^T & 0 \end{bmatrix}$ and det $[\,\cdot\,]$ denotes the determinant of the corresponding matrix.

If the solution vector $\underline{X}$ of (4.30) is $\hat{\underline{X}}$ then simple manipulations in (4.29) using (4.31) yield

$$V(\hat{\underline{X}}) = \frac{2^{n-1}\det[P]\,k_i^2}{\det[H]} .$$

The minimum of all such minima on $\mathbb{B}$ yields ε that defines the region of stability. A similar expression was obtained by Weissenberger [67] for the stability domains in the case of a system with a single nonlinearity.

Case (iii) Quadratic part neglected (i.e. $P \equiv 0$):

In some situations it is possible to neglect the contribution of the quadratic $\underline{X}^T P\,\underline{X}$ compared to the

integrals of the nonlinearity when the state variables assumed values near the point where the sector violation occurs. In such cases it is possible to avoid the solution of the (n+m) nonlinear equations of (4.21) as shown below. Making $P \equiv 0$, the V-function in (4.10) takes the form

$$V(\underline{X}) = \sum_{i=1}^{m} \beta_i \int_0^{\sigma_i} f_i(s_i)\, ds_i \tag{4.33}$$

and equation (4.21) reduces to

$$\begin{bmatrix} 0 & -C \\ C^T & 0 \end{bmatrix} \begin{bmatrix} \underline{X} \\ \underline{\tau} \end{bmatrix} = \begin{bmatrix} -C\ \beta\ \underline{f}(\underline{\sigma}) \\ \underline{\sigma} \end{bmatrix} \tag{4.34}$$

from which we obtain

$$C\ \beta\ \underline{f}(\underline{\sigma}) = C\,\underline{\tau}$$

or $$C(\beta\ \underline{f}(\underline{\sigma}) - \underline{\tau}) = 0 \tag{4.35}$$

The trivial solution of (4.35) yields

$$\beta\ \underline{f}(\underline{\sigma}) = \underline{\tau} = [0,\ 0\ ..\ \tau_i\ ..\ 0] \tag{4.36}$$

Equation (4.35) implies

$$f_j(\sigma_j) = 0 \quad , \ j = 1,2,\ldots m;\ j \neq i$$

or $$\sigma_j = 0 \quad , \ j = 1,2,\ldots m;\ j \neq i\ . \tag{4.37}$$

This implies that all σ_j's, $(j = 1,2,\ldots m,\ j \neq i)$ are zero. Also since the ith component $\hat{\sigma}_i$ of $\underline{\sigma}$ is a priori specified in (4.17), the ith equation of (4.35) yields τ_i according to

$$\tau_i = \beta_i\ f_i(\hat{\sigma}_i) \tag{4.38}$$

Thus the solution vector $\hat{\underline{\sigma}}$ of $\underline{\sigma}$ is given by

$$\hat{\underline{\sigma}} = [\ 0,\ 0,\ \ldots\ k_i,\ 0,\ \ldots\ 0]^T \tag{4.39}$$

and may be directly substituted in (4.24) to obtain a minimum of $V(\underline{X})$ on the hyperplane given by (4.17). Equation (4.39) thus defines precisely a point $\hat{\underline{X}}$ on $\mathbb{B}$ at which one of the σ_i's reaches its limit according to (4.9) while all others assume zero values. We shall define such points on $\mathbb{B}$ as central points.

The minimum of all the V-values computed at all the central points on $\mathbb{B}$ thus yields ε that defines the region of attraction according to (4.11).

4.3 STABILITY REGIONS FOR MULTIMACHINE POWER SYSTEMS:

4.3.1 Problem formulation:

In this section a method for computing the transient stability regions for multimachine power systems based on the results of the previous section is proposed. This method utilizes the sector violation properties of the nonlinearities of the power system state models. In Section 2.6 of Chapter II state models of the form

$$\dot{\underline{X}} = A\ \underline{X} - B\ \underline{f}(\underline{\sigma}) \tag{4.40a}$$

$$\underline{\sigma} = C^T\ \underline{X} \tag{4.40b}$$

have been constructed in the minimal state space of (2n-1) for the non-uniform damping and (2n-2) for the

uniform damping cases. Thus A, B and C^T constitute a minimal realization of the transfer function matrices $W_U(s)$ and $W_N(s)$ given in equation (2.60) and (2.63) of Chapter II. For such systems Lyapunov functions of the type

$$V(\underline{X}) = \underline{X}^T P \underline{X} + \sum_{i=1}^{m} \beta_i \int_0^{\sigma_i} f_i(s_i) ds_i \qquad (4.41)$$

have been constructed in reference [10], where P is a positive definite symmetric matrix and β_i a constant. These Lyapunov functions are used in the analysis to follow. The procedure of constructing such Lyapunov functions is based on a theorem due to Moore and Anderson [81]. This theorem is stated and the results of its application in the derivation of the V-function (4.41)are shown in Appendix C.

The nonlinearities $f_i(\sigma_i)$, i = 1,2,...m occurring in power system models (4.40) do not satisfy the Popov's sector conditions (4.8) over the entire state space. In fact from (2.38) σ_i is known to violate this sector at

$$\begin{aligned} k_i &= \pi-2(\delta_p^o - \delta_q^o) \\ \text{and} \quad \ell_i &= \pi+2(\delta_p^o - \delta_q^o) \end{aligned} \qquad (4.42)$$

as shown in Figure 4.2. Therefore the Lyapunov function $V(\underline{X})$ satisfies the necessary sign definite properties for only those σ_i, i = 1,2,...m that lie in the closed interval

$$\sigma_i \in [\pi-2(\delta_p^o-\delta_q^o),\ -\pi-2(\delta_p^o-\delta_q^o)] \qquad (4.43)$$

Hence a region of asymptotic stability is defined for all

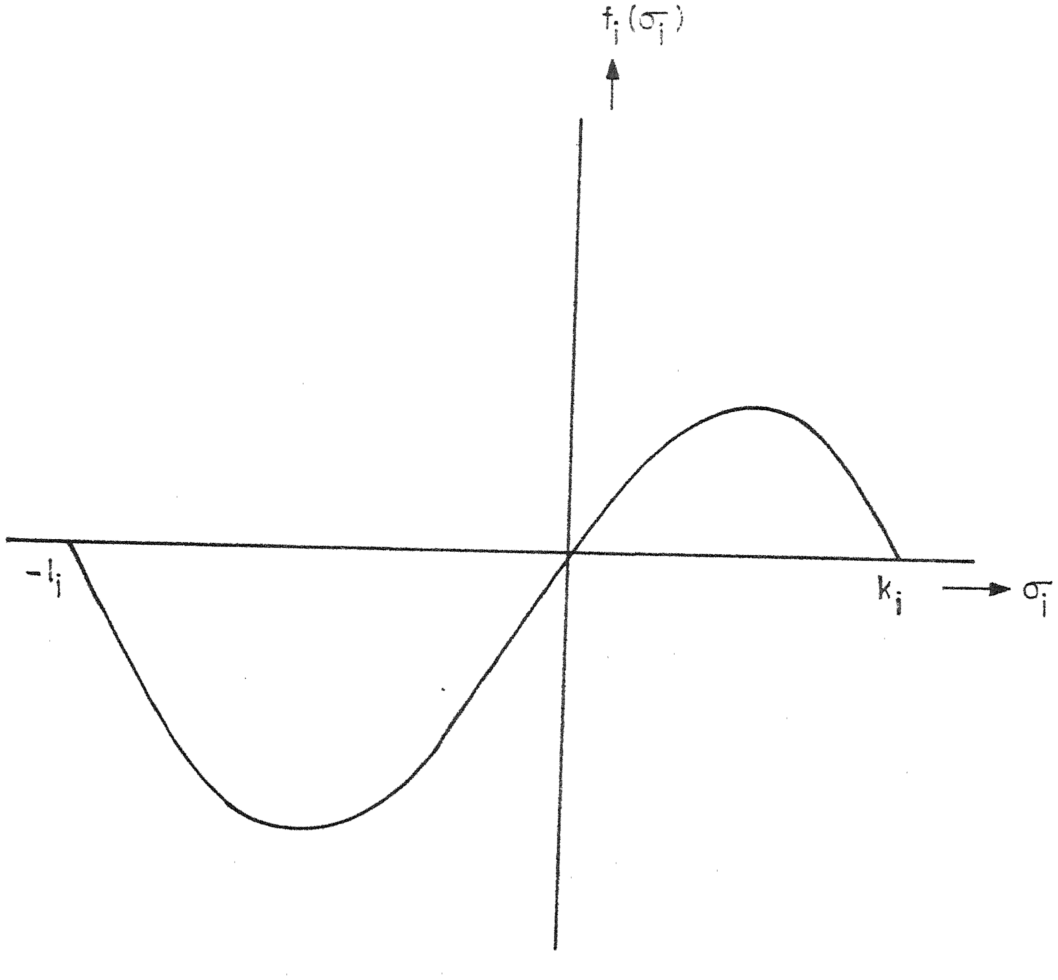

FIG. 4.2: THE POWER SYSTEM NONLINEARITY

σ_i lying in the open interval

$$\sigma_i \in (\pi - 2(\delta_p^o - \delta_q^o), -\pi - 2(\delta_p^o - \delta_q^o)) . \tag{4.44}$$

In this region the time derivative $\dot{V}(\underline{X})$ is given by [10]

$$\dot{V}(\underline{X}) = -\underline{X}^T L\ L^T\ \underline{X} - 2\ \alpha\ f^T(\underline{\sigma})\underline{\sigma} \tag{4.45}$$

for all $g_i = \infty$ (infinite Popov sector). LL^T is obtained from the Lyapunov matrix equation

$$A^T P + P A = - L L^T \tag{4.46}$$

An estimate of this stability domain will now be obtained utilizing the results of the previous section.

4.3.2 Determination of stability regions:

The procedure of determining the stability region of the power system is slightly involved because of the fact that not all the σ_i's are linearly independent. In fact, examination of equation (4.40b) reveals that only (n-1) components of the m-vector $\underline{\sigma}$ are linearly independent. The remaining (m-n+1) components can be obtained as a linear combination of these linearly independent variables [24]. Hence the boundary of the polyhedron $\mathbb{B}$ defined in Section 4.2 is uniquely defined by these linearly independent variables. As an example consider a 4-machine system. Equation (4.40) gives

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} [\underline{X}] \tag{4.47}$$

in which the coefficient matrix has only 3 linearly independent rows. Thus the rank of C^T is only 3. This implies that only 3 σ_i's are linearly independent. For instance, σ_1, σ_2 and σ_3 can be assumed to be linearly independent. Then the other variables are given by

$$\begin{aligned} \sigma_4 &= \sigma_2 - \sigma_1 \\ \sigma_5 &= \sigma_3 - \sigma_1 \end{aligned} \tag{4.48}$$

and $\sigma_6 = \sigma_3 - \sigma_2$.

In view of this property, equation (4.40b) can be partitioned in the form

$$\begin{bmatrix} \underline{\sigma}_I \\ \underline{\sigma}_D \end{bmatrix} = \begin{bmatrix} C_I^T \\ C_D^T \end{bmatrix} \underline{X} \tag{4.49}$$

where $\underline{\sigma}_I$ and $\underline{\sigma}_D$ are the linearly independent and dependent subvectors respectively of $\underline{\sigma}$. The matrices C_I and C_D are the properly partitioned submatrices of C. The choice of the linearly independent set of σ_i's however is not unique. Any (n-1) of the n variables of $\underline{\sigma}$ whose linear combination results in the components of $\underline{\sigma}_D$ can, infact, be chosen. Again, as an example, consider the 4-machine case. From (4.47) one can choose σ_1, σ_2 and σ_5 as the linearly independent set. Then the other three variables σ_3, σ_4 and σ_6 are derived to be

$$\sigma_3 = \sigma_1 + \sigma_5$$

$$\sigma_4 = \sigma_2 - \sigma_1 \qquad (4.50)$$

and $\sigma_6 = \sigma_1 - \sigma_2 + \sigma_5$

The number of such sets of $\underline{\sigma}_I$'s has an upper bound of ${}^mC_{n-1}$. Each of these $\underline{\sigma}_I$'s thus uniquely defines a polyhedron $\mathbb{B}$ in the state space. Thus an upper bound on the possible number of polyhedrons is also ${}^mC_{n-1}$.

Now consider one such linearly independent $\underline{\sigma}_I$ and define (2n-2) hyperplanes in the following way:

$$(C_I^T \underline{X})_i = \pi - 2(\delta_p^o - \delta_q^o) \ , \quad i=1,2,\ldots(n-1)$$

$$\text{and } (C_I^T \underline{X})_j = -\pi - 2(\delta_p^o - \delta_q^o), \quad j = 1,2,\ldots(n-1) \qquad (4.51)$$

These equations, exactly identical to those defined in (4.16), define the boundary of the (n-1) dimensional closed surface $\mathbb{B}$, $\{\underline{X}: \pi-2(\delta_p^o-\delta_q^o) \leq (\underline{\sigma}_I)_i \leq -\pi-2(\delta_p^o-\delta_q^o)\}$. This convex polyhedron $\mathbb{B}$ has (2n-2) central points. Let the coordinates of that central point at which minimum of $V(\underline{X})$ occurs be $\hat{\underline{X}}$. Then following the procedure discussed in case (iii) of section (4.2.3), under the assumption that the contribution due to quadratic part is negligible, $V(\hat{\underline{X}})$ is given by

$$V(\hat{\underline{X}}) = \hat{\underline{\sigma}}_I^T (C_I^T P^{-1} C_I)^{-1} \hat{\underline{\sigma}}_I + \sum_{i=1}^{m} \beta_i \int_o^{\sigma_i} f_i(s_i) ds_i \qquad (4.52)$$

where $\hat{\underline{\sigma}}_I = C_I^T \hat{\underline{X}}$.

The minimum of all such V-values obtained on all polyhedrons yields the value of ε that defines the region of stability according to equation (4.11). Thus the calculation of $V(\underline{X})$ at a maximum of ${}^{m}C_{n-1}$ (2n-2) central point (120 points in a 4-machine case) is required to obtain ε.

For practical systems such enumeration and computation is difficult when the number of machines becomes large . We shall now discuss a procedure that reduces this computational complexity. One of the machines say 'r' is chosen at a time and the linearly independent variables $(\underline{\sigma}_I)_i$, i = 1,2,...(n-1), are selected in such a way that the corresponding rotor angular difference $(\delta_p^o - \delta_q^o)$ involve the angles of this machine 'r', i.e., δ_p^o or $\delta_q^o = \delta_r^o$. This is equivalent to taking machine 'r' as reference. We are also assured that such a choice of $(\underline{\sigma}_I)_i$ results in a linearly dependent set $\underline{\sigma}_D$. Since there are n machines in all, there will be 'n' polyhedrons each having its boundary $\mathbb{B}$ containing (2n-2) central points. Thus there will be a total of n(2n-2) central points at which $V(\underline{X})$ has to be computed. Moreover, at certain of these points the components of $\underline{\sigma}_D$ obtained from the corresponding $\underline{\sigma}_I$ may violate their respective sector conditions of the form (4.9). Such points are discarded since $V(\underline{X})$ may

not satisfy the properties of a Lyapunov function. The minimum of the V-values computed at all the valid central points yields an estimate of ε.

The method discussed above is now summed up in the form of an algorithm for practical implementation as follows:

ALGORITHM 1:

Step 1: Obtain the stable equilibrium point of the post-fault system $(\delta_1^o-\delta_j^o)$, $j=2,3,\ldots n$. This gives $(n-1)$ angular differences.

Step 2: Using the stable equilibrium point in step 1, compute all other angular differences $(\delta_p^o-\delta_q^o)$ such that $p< q$. This gives an additional set of $(m-n+1)$ values. With the values of angular differences in step 1, there are a total of 'm' differences.

Step 3: Choose machine 'r'. Set $r=1$.

Step 4: From the total of 'm' angular differences in step 2, obtain all the $(n-1)$ angular differences $(\delta_p^o-\delta_q^o)$ such that p or $q = r$. Using these values compute $k_i = \pi-2(\delta_p^o-\delta_q^o)$ and $\ell_i = \pi+2(\delta_p^o-\delta_q^o)$, $i=1,2,\ldots(n-1)$.

Step 5: Using the values of k_i and ℓ_i of step 4, obtain the coordinates of all central points given by $(0, 0, \ldots, k_i \text{ or } -\ell_i, 0, 0)$, $i=1,2,\ldots(n-1)$.

This gives a total of (2n-2) central points. Now each of these central points represents one set of $\underline{\sigma}_I$.

Step 6: Compute $\underline{\sigma}_D$ for each of the central points in step 5 and check for violations of sector cconditions in the elements of $\underline{\sigma}_D$. If there are no violations compute $V(\underline{X})$. Otherwise discard the central point.

Step 7: Obtain the minimum of all $V(\underline{X})$ computed in step 6.

Step 8: Choose the next machine by incrementing in r (i.e. $r = r+1$) and go to step 4. If $r=n+1$ go to step 9.

Step 9: Choose the minimum of all the minima obtained in step 7. This defines ε.

The algorithm was implemented on several systems and the results of study on a 5-machine system are presented in section 'Results of Study'.

4.3.3 Comparison of the method with that of reference [31]:

The technique of estimating the stability region detailed above will now be compared with that of reference [31]. For this to be possible, it is necessary to study the physical significance associated with a central point. As mentioned earlier, at this point on $\mathbb{B}$

all the linearly independent variables $(\underline{\sigma}_I)_i$ $(i=1,2\ldots(n-1))$ of $\underline{\sigma}_I$ excepting one assume zero value. The nonzero component assumes a value of $\pi-2(\delta_p^o-\delta_q^o)$ or $-\pi-2(\delta_p^o-\delta_q^o)$. For example consider the 4-machine case. The linearly independent set $\underline{\sigma}_I$ with machine 1 as reference is

$$
\begin{aligned}
(\underline{\sigma}_I)_1 &= (\delta_1-\delta_2)-(\delta_1^o-\delta_2^o) \\
(\underline{\sigma}_I)_2 &= (\delta_1-\delta_3)-(\delta_1^o-\delta_3^o) \\
\text{and} \quad (\underline{\sigma}_I)_3 &= (\delta_1-\delta_4)-(\delta_1^o-\delta_4^o) \quad .
\end{aligned}
\tag{4.53}
$$

With this set one possible central point is

$$
\begin{aligned}
(\delta_1-\delta_2) - (\delta_1^o-\delta_2^o) &= \pi-2(\delta_1^o-\delta_2^o) \\
(\delta_1-\delta_3) - (\delta_1^o-\delta_3^o) &= 0 \\
\text{and} \quad (\delta_1-\delta_4) - (\delta_1^o-\delta_4^o) &= 0 \quad .
\end{aligned}
\tag{4.54}
$$

Consider the system on no load so that $\delta_i^o = 0$, $i=1,2,3,4$. Then with (4.54) one obtains $\delta_1=0$ (since it is the reference), $\delta_2 = -\pi$, $\delta_3 = 0$ and $\delta_4 = 0$. This situation implies that the value of $V(\underline{X})$ computed at this point corresponds to the potential energy **required for taking the** machine 2 out of step from the rest of the system. Thus the system while losing synchronism is divided into two groups one of which contains machine 2 and other contains the rest [17,31]. This analysis applies to every other point. It is precisely on this basis that the authors of reference [31]

approximated the unstable equilibrium points. It may therefore be concluded that the method proposed in Section 4.3.2 provides a necessary theoretical rationale for the development in reference [31] with the following additional features:

1. The search among the $n(2n-2)$ central points of 'n' polyhedrons takes into account the machine reference automatically.

2. An estimate of ε is obtained without violations of the sector conditions of any of the m nonlinearities.

The method, however, is found to yield slightly conservative results compared to those of reference [31].

4. 4.3.4 An alternate procedure of estimating the region of stability:

An alternate procedure utilizing the sector violation properties of the power system nonlinearities that yields better stability domains is suggested here. Considering the fact that only the first $(n-1)$ variables of $\underline{\sigma}$ with machine 1 as reference are linearly independent, it was conjectured in reference [24], that the minimum of $V(\underline{X})$ occurs at one of the vertices of an $(n-1)$ dimensional polyhedron in the σ-space defined by this set of linearly independent variables according to equation (4.51). But the fact that any $(n-1)$ components of $\underline{\sigma}$ can be chosen to be linearly independent as described

earlier was overlooked. Using this property it is now possible to estimate the region of stability as follows:

Consider a polyhedron defined with machine 'r' as reference, i.e. either δ_p^o or $\delta_q^o = \delta_r^o$ in the set (4.51). This polyhedron has 2^{n-1} vertices. It can be shown that each of these points correspond to the machine 'r' going out of step with respect to the rest of the machines. For example, consider equations (4.53) of the 4-machine case where machine 1 is the reference. The polyhedron $\mathbb{B}$ formed by the 3 linearly indepdndent variables $(\underline{\sigma}_I)_i$, i=1,2,3 has 8 vertices. Also assume an unloaded case such that $\delta_i^o = 0$, $i = 1,2,3,4$. Then the coordinates of the vertices of $\mathbb{B}$ are (π, π, π), $(\pi, -\pi, \pi)$ and so on. Each of these vertices therefore, corresponds to the machines 2,3 and 4 going out of step with respect to machine 1. Equivalently machine 1 looses synchronism. Moreover, the linearly dependent variables can be verified to be taking value of 2π or 0. This implies that the dependent variables are pinned down at the origin of their respective σ-$f(\sigma)$ planes since the solutions are periodic in 2π.

In a loaded system, however, the minimum of the values of $V(\underline{X})$ computed at the 8 vertices of $\mathbb{B}$ yields the minimum energy required to dissociate machine 'r' from the system. Similar analysis holds for the other machines **also**. Since there are n machines in all, the minimum of all the

V-values obtained on all the polyhedrons yields an estimate of ε that defines the region of stability.

In view of the above analysis, the following algorithm is proposed.

ALGORITHM 2:

Step 1: Obtain the stable equilibrium point of the post-fault system $(\delta_1^o-\delta_j^o)$, $j = 2,3,\ldots n$.

Step 2: Calculate all possible angular differences $(\delta_p^o-\delta_q^o)$, $p,q = 1,2,\ldots n$ $(p< q)$. The total number of such differences is m $(= n(n-1)/2)$.

Step 3: Choose machine 'r'. Set with r = 1. It is assumed here that the machine 1 goes out of step.

Step 4: Among the differences of step 2, pick all the (n-1) differences $(\delta_p^o-\delta_q^o)$ such that p or q = r.

Step 5: Compute the values of k_i and ℓ_i for each of the (n-1) differences in step 4.

Step 6: Choose $(\underline{\hat{\sigma}}_I)_i = k_i$ if $k_i < \ell_i$

$= -\ell_i$ if $\ell_i < k_i$, $i=1,2,\ldots(n-1)$

and form the vector $\underline{\hat{\sigma}}_I$.

Step 7: Set all the (m-n+1) linearly dependent variables at $-2(\delta_p^o-\delta_q^o)$, and form the vector $\underline{\hat{\sigma}}_D$.

Step 8: Compute $V(\underline{X})$. Set r=r+1 and go to step 4 if all the machines are not exhausted. If r = n+1 go to step 9.

Step 9: Choose the minimum of all V-values obtained in step 8. This gives ε.

This method was implemented on a number of systems and has been found to give highly accurate results. Further, the method avoids the problem of machine reference as in method I and the value of $V(\underline{X})$ for a particular machine loosing synchronism is uniquely determined. Results of study on a 5-machine system along with a comparison with the other methods are given in the next section.

4.4 RESULTS OF STUDY:

The system chosen for study is a 5-machine system the data of which is given in Appendix D. The magnitudes of the voltages behind the transient reactances, mechanical power inputs and inertia constants are given in Table 4.1. A fault at bus 6 is cleared by isolating line 13. The

Table 4.1: System Data

Machine i	Voltage behind transient react. (E_i)	Mech.power input (P_{mi})	Inertia constant (H_i)
1	1.05734	0.900	2.30
2	1.08650	1.750	10.30
3	1.08321	3.200	12.72
4	1.05386	3.500	27.00
5	0.93777	0.443	∞

driving and transfer admittances between the internal nodes of the machines are calculated and shown in Tables 4.2 and 4.3 for the faulted and the post-fault systems respectively. Transfer conductances are neglected for the post-fault state but the short circuit conductances are readjusted to satisfy equation (2.14) of Chapter II. All data are in p.u.

Table 4.2(a): G-matrix for the faulted system.

M/C No.	1	2	3	4	5
1	1.17664	-0.00360	0.0000	-0.07078	-0.50932
2	-0.00360	0.25677	0.0000	0.08380	-0.00924
3	0.00000	0.00000	0.0000	0.00000	0.00000
4	-0.07078	0.08380	0.0000	3.42242	-0.27819
5	-0.50932	-0.00924	0.0000	-0.27819	2.36496

Table 4.2(b): B-matrix for the faulted system.

M/C No.	1	2	3	4	5
1	-4.52778	0.28854	0.0000	-0.94978	1.86827
2	0.28854	-3.42500	0.0000	0.80360	0.50746
3	0.00000	0.00000	-13.9470	0.00000	0.00000
4	-0.94978	0.80360	0.0000	-10.88623	3.88867
5	1.86827	0.50746	0.0000	3.88867	-8.80014

Table 4.3(a): G-matrix for the post-fault system

M/C No.	1	2	3	4	5
1	1.17877	0.01445	0.04455	-0.03210	-0.50658
2	0.01445	0.29015	0.15703	0.16609	0.02151
3.	0.04455	0.15703	1.33232	0.35635	0.07116
4	-0.03210	0.16609	0.35635	3.62085	-0.21297
5	-0.50658	0.02151	0.07116	-0.21297	2.36799

Table 4.3(b): B-matrix for the post-fault system

M/C No.	1	2	3	4	5
1	-4.42707	0.38725	0.71489	1.22917	2.04542
2	0.38725	-3.33079	0.69596	1.07222	0.68124
3	0.71489	0.69596	-5.16262	1.97359	1.25780
4	1.22917	1.07222	1.97359	-10.12185	4.38045
5	2.04542	0.68124	1.25780	4.38045	-8.56853

The post-fault equilibrium state (with the transfer conductances neglected) is given by

$$\delta_1^o-\delta_2^o = -0.34147$$

$$\delta_1^o-\delta_3^o = -0.19861$$

$$\delta_1^o-\delta_4^o = 0.04049$$

and $\delta_1^o-\delta_5^o = 0.08604$.

Damping is assumed to be uniform and λ is chosen to be unity. Then the swing equations describing the system in the post-fault state with machine 1 as reference are given by

$$\frac{d^2}{dt^2}(\delta_1-\delta_j) + \frac{d}{dt}(\delta_1-\delta_j) = \pi f\Big[\frac{1}{H_1}\{(P_{m1}-E_1^2\ G_{11}) - \sum_{i=2}^{5} E_1E_iB_{1i}\ \sin(\delta_1-\delta_i)\} - \frac{1}{H_2}\{ (P_{mj}-E_j^2G_{jj}) - \sum_{\substack{i=1\\ i\neq j}}^{5} E_jE_iB_{ji}\ \sin(\delta_j-\delta_i)\}\Big],\ j=2,3,4,5 \quad (4.55)$$

The parameters in (4.55) are obtained from Tables 4.1 and 4.3. When these equations are cast in the Lure' form, the number of nonlinearities $f_i(\sigma_i)$ is 10. They are

$$f_1(\sigma_1) = [\sin(\sigma_1 + (\delta_1^o-\delta_2^o)) - \sin(\delta_1^o-\delta_2^o)]$$

$$f_2(\sigma_2) = [\sin(\sigma_2 + (\delta_1^o-\delta_3^o)) - \sin(\delta_1^o-\delta_3^o)]$$

$$f_3(\sigma_3) = [\sin(\sigma_3 + (\delta_1^o-\delta_4^o)) - \sin(\delta_1^o-\delta_4^o)]$$

$$f_4(\sigma_4) = [\sin(\sigma_4 + (\delta_1^o-\delta_5^o)) - \sin(\delta_1^o-\delta_5^o)]$$

$$f_5(\sigma_5) = [\sin(\sigma_5 + (\delta_2^o-\delta_3^o)) - \sin(\delta_2^o-\delta_3^o)]$$

$$f_6(\sigma_6) = [\sin(\sigma_6 + (\delta_2^o-\delta_4^o)) - \sin(\delta_2^o-\delta_4^o)]$$

$$f_7(\sigma_7) = [\sin(\sigma_7 + (\delta_2^o-\delta_5^o)) - \sin(\delta_2^o-\delta_5^o)]$$

$$f_8(\sigma_8) = [\sin(\sigma_8 + (\delta_3^o-\delta_4^o)) - \sin(\delta_3^o-\delta_4^o)]$$

$$f_9(\sigma_9) = [\sin(\sigma_9 + (\delta_3^o-\delta_5^o)) - \sin(\delta_3^o-\delta_5^o)]$$

$$f_{10}(\sigma_{10}) = [\sin(\sigma_{10}+(\delta_4^o-\delta_5^o))- \sin(\delta_4^o-\delta_5^o)] .$$

These nonlinearities violate their corresponding sectors at k_i and $-\ell_i$ given in Table 4.4.

Table 4.4: Sector violation information

Sl.No.	σ_i	Value of $(\delta_i^o-\delta_j^o)$	$k_i = \pi-2(\delta_i^o-\delta_j^o)$	$-\ell_i = -\pi - 2(\delta_i^o - \delta_j^o)$
1	σ_1	-0.34147	3.48306	-2.80012
2	σ_2	-0.19861	3.34020	-2.94298
3	σ_3	0.04049	3.10110	-3.18238
4	σ_4	0.08604	3.05555	-3.22763
5	σ_5	0.14286	2.99873	-3.28445
6	σ_6	0.38196	2.75963	-3.52355
7	σ_7	0.42751	2.71408	-3.56910
8	σ_8	0.23910	2.90249	-3.38069
9	σ_9	0.28465	2.85694	-3.42624
10	σ_{10}	0.04555	3.09604	-3.18714

Here four of the ten nonlinearities are linearly independent. Algorithm 1 will now be implemented.

Choosing the machine 1 ($r = 1$) first, the linearly independent variables selected are σ_1, σ_2, σ_3 and σ_4. With these as components of $\underline{\sigma}_I$, one central point is obtained with the following values:

$$\sigma_1 = -2.80012; \quad \sigma_2 = 0 \; ; \; \sigma_3 = 0 \quad \text{and} \quad \sigma_4 = 0.$$

This point corresponds to machine 2 going out of step and machine 1 is the reference. With this set constituting $\underline{\sigma}_I$, the components of $\underline{\sigma}_D$ are :

$$\sigma_5 = 2.80012 \; ; \quad \sigma_6 = 2.80012 \; ; \; \sigma_7 = 2.80012$$

$$\sigma_8 = 0.0 \; ; \quad \sigma_9 = 0.0 \text{ and } \quad \sigma_{10} = 0.0 \; .$$

With these, $V(\underline{X})$ is calculated to be $= 3.42169$.

It may, however, be noted that σ_6 and σ_7 violate their sector conditions. According to the algorithm, this point is inadmissible. Table 4.5 shows such an analysis at the other central points. Only 4 points have been shown because the rest four points on this polyhedron yield larger values. Similar procedure is adopted for other machines 'r', r = 2,3,4 and 5. A summary of this analysis is shown in Table 4.6 where the values of $V(\underline{X})$ for the 5 machines going out of step and with every machine selected as reference are given. Also

the sector violation conditions are indicated. The minimum of all the V-values at all the admissible central points is 3.18156. Hence the stability boundary is defined by

$$V(\underline{X}) = 3.18156 \text{ .}$$

Table 4.5: Choice of M/c 1: An analysis of central points.

Sl.No.	σ_1	σ_2	σ_3	σ_4	Value of V(X)	Sector viol.
1	-2.80012	0.0	0.0	0.0	3.42169	Yes
2	0.0	-2.94298	0.0	0.0	7.50162	Yes
3	0.0	0.0	3.10110	0.0	15.71059	Yes
4	0.0	0.0	0.0	3.05555	13.87118	Yes

Now the procedure indicated in algorithm 2 will be adopted. Accordingly we first choose machine 1 (r=1). Hence the components of $\underline{\sigma}_I$ are σ_1, σ_2, σ_3 and σ_4 and are assigned values

$$\sigma_1 = -2.80012 \quad ; \quad \sigma_3 = 3.1011$$
$$\sigma_2 = -2.94980 \quad ; \quad \sigma_4 = 3.05555 \text{ .}$$

Assign values of $-2(\delta_k^o-\delta_j^o)$ for all the other linearly dependent variables. Thus

$$\sigma_5 = -0.28572 \quad ; \quad \sigma_8 = -0.47820$$
$$\sigma_6 = -0.76392 \quad ; \quad \sigma_9 = -0.56930$$
$$\sigma_7 = -0.85602 \quad ; \quad \sigma_{10} = -0.09110$$

Table 4.6: Effect of machine references.

Sl. No.	Ref. M/c	Variables of $\underline{\sigma}_I$	Machines loosing synchronism 1	2	3	4	5
1	1	σ_1	-	-2.80012	0.0	0.0	0.0
2		σ_2	-	0.0	-2.94298	0.0	0.0
3		σ_3	-	0.0	0.0	3.10110	0.0
4		σ_4	-	0.0	0.0	0.0	3.05555
	Value of $V(\underline{X})$		-	3.42169 (VL)	7.50162 (VL)	15.71059 (VL)	13.87118 (VL)
5	2	σ_1	-2.80012	-	0.0	0.0	0.0
6		σ_5	0.0	-	2.99873	0.0	0.0
7		σ_6	0.0	-	0.0	2.75963	0.0
8		σ_7	0.0	-	0.0	0.0	2.71408
	Value of $V(\underline{X})$		8.75572 (VN)	-	12.8044 (VN)	15.58135 (VN)	13.73607 (VN)
9	3	σ_2	-2.94298	0.0	-	0.0	0.0
10		σ_5	0.0	2.99873	-	0.0	0.0
11		σ_8	0.0	0.0	-	2.90249	0.0
12		σ_9	0.0	0.0	-	0.0	2.85694
	Value of $V(\underline{X})$		8.90831 (VL)	3.18156 (VN)	-	15.7627 (VL)	13.89756 (VL)
13	4	σ_3	3.1011	0.0	0.0	-	0.0
14		σ_6	0.0	2.75963	0.0	-	0.0
15		σ_8	0.0	0.0	2.90249	-	0.0
16		σ_{10}	0.0	0.0	0.0	-	3.09604
	Value of $V(\underline{X})$		9.46432 (VL)	3.45545 (VL)	7.52921 (VL)	-	13.81387 (VL)
17	5	σ_4	3.05555	0.0	0.0	0.0	-
18		σ_7	0.0	2.71408	0.0	0.0	-
19		σ_9	0.0	0.0	2.85694	0.0	-
20		σ_{10}	0.0	0.0	0.0	3.09604	-
	Value of $V(\underline{X})$		9.44373 (VN)	3.48729 (VN)	7.55031 (VN)	20.89913 (VN)	-

Note: VL : indicates sector violations.
VN : indicates no sector violations.

With these variables $V(\underline{X}) = 4.03076$.

The procedure is followed in a similar manner for the other machines i.e., $r = 2,3,4,$and 5. The V-values obtained at the corresponding corner point are given in Table 4.7. The minimum of the V-values at these points yields ε.

Table 4.7: Corner point evaluation.

Sl. No.	M/c going out of step	Linearly independent variables to be chosen	Value of $V(\underline{X})$
1	1	$\sigma_1 = -2.80012$; $\sigma_3 = 3.10110$ $\sigma_2 = -2.94298$; $\sigma_4 = 3.05555$	9.68689
2	2	$\sigma_1 = -2.80012$; $\sigma_6 = 2.75963$ $\sigma_5 = 2.99873$; $\sigma_7 = 2.71408$	4.03076
3	3	$\sigma_2 = -2.94298$; $\sigma_8 = 2.90249$ $\sigma_5 = 2.99873$; $\sigma_9 = 2.85694$	7.66021
4	4	$\sigma_3 = 3.10110$; $\sigma_8 = 2.90249$ $\sigma_6 = 2.75963$; $\sigma_{10} = 3.09604$	15.60612
5	5	$\sigma_4 = 3.05555$; $\sigma_8 = 2.90249$ $\sigma_7 = 2.71408$; $\sigma_{10} = 3.09604$	14.84772

A comparative study of the various methods including that of reference [31] will now be made.

Table 4.8 gives the values of the V-function and the corresponding estimate of the critical clearing times obtained. The actual critical clearing time is also obtained by integrating the differential equations of the type (4.55) in the faulted state. The parameters in these equations assume values given in Tables 4.1 and 4.2. The critical clearing time computed, is also shown in Table 4.8.

Table 4.8: A comparative study.

	Values of $V(\underline{X})$ computed using method of			
	ref.[31]*	Algorithm 1	Algorithm 2	Actual integration
Value of $V(\underline{X})$	3.48729	3.18156	4.03076	-
Critical clearing time sec.	0.1780	0.173	0.190	0.205

* Choosing the machine with the largest inertia as reference.

It is observed that the critical clearing time computed using the results of algorithm 2, yields clearing times close to the actual, than those offered by the technique of reference [31] and the algorithm 1 suggested in this chapter.

4.5 CONCLUSIONS:

In this chapter regions of attraction for multi-nonlinear systems using the sector violation properties

of the nonlinearities have been derived. These results constitute a generalization of those obtained in the literature for systems with a single nonlinearity. This generalization involves the solution of a set of nonlinear equations in a general case. It is also shown that explicit expression is possible in special cases. These results are extended to the multimachine power system stability problem and it has been shown that they provide a theoretical basis for a method suggested by Prabhakara and El-Abiad [31]. However, this method is suitably modified by considering the sector violations of the dependent nonlinearities and machine reference. The procedure is summarized in algorithm 1. Algorithm 2 is new. Both of these have been implemented on a number of practical systems [79]. Results of study on a 5-machine system are given and the methods are compared.

CHAPTER V

COHERENCY IDENTIFICATION AND DYNAMIC EQUIVALENTS USING ENERGY FUNCTIONS

5.1 INTRODUCTION:

This chapter deals with the development of reduced order models of power systems for use in the transient stability of large scale power systems. Due to the increased interconnections in power systems, it usually happens that the behaviour of one section of the system may not affect significantly the performance of some other section of the system under study. The stability analysis of this latter section called the 'study system' can then be carried out by a simplified representation of the former section called the 'external area' both in size and in detail. This procedure reduces the burden on the computer memory and saves considerable computation time. The reduced order model of the external system thus obtained is called the 'Dynamic equivalent'.

The development of dynamic equivalents for large power systems for use in stability analysis, however, is not as easy as for load flow studies [80]. Some of the difficulties associated with equivalencing are as follows [55]:

(i) The dynamical nature of the synchronous machines, governors etc. cannot be easily represented on the equivalent.

(ii) A proper identification of the portion of the system to be equivalenced is required,

and (iii) Inertia allocation among the equivalenced machines for multimachine representation poses formidable problems.

Currently, three different approaches exist for obtaining the dynamic equivalent for a large power system. These are

(i) Use of network equivalents and inertia allocation [54, 55],

(ii) System reduction employing the property of coherency among machines[53,56,57,59],

and (iii) Modal techniques [61-64].

Of these, coherency based dynamic equivalents are by far the simplest. This has a potential application as a production tool in planning and represents a key step in the on line monitoring techniques [57]. The method consists in identifying the machines that swing together during a transient disturbance by processing the swing curves. Pairs of generators outside the study area that tend to swing together are compared and generators are classified as coherent if the maximum excursion in their angular difference does not exceed a pre-assigned limit[53]. This concept of coherency has been extended to generator as well as non-generator buses of the power system [56].

Two buses are defined as coherent if for all time (i) the ratio of voltage magnitudes of the buses is constant and (ii) the difference in voltage angles of the two buses is constant. A method for coherency identification without actually conducting a stability study has been suggested by Lee and Schweppe [59]. They utilize the concept of 'a distance measure' to ascertain the importance of a generator to the stability of the machines in the study area and propose a set of 'features' a term used in pattern recognition, for identifying coherent machines. Once coherency among the machines is established, there remains the problem of deriving the dynamic equivalent. Chang and Adibi [53] have used the well known node elimination formula. This technique alters every element of the admittance matrix with all the internal nodes of generators retained. A simple computational procedure based on the power invariance criterion has been proposed by DeMello etal [56,57]. Here only those elements of the admittance matrix that correspond to the coherent group are altered.

In this chapter energy function is proposed as a criterion for coherency recognition as well as for deriving the dynamic equivalent. These energy functions have been used as Lyapunov functions in the transient stability investigations of power systems. The reduced order dynamic equivalent derived by using this criterion

is useful in the stability analysis of the unreduced system via the Lyapunov methods. There are also advantages of such an analysis in the evaluation of the unstable equilibrium points of the post-fault state of the system as pointed out in a recent paper by Lüders [17]. .

A useful step in obtaining the dynamic equivalent is to divide the system into the following two areas:

(i) the faulted area or usually referred to as the study area in which the system behaviour is of direct interest. This is usually specified a priori,

and (ii) the external area or the remainder of the system.

The dynamic equivalent is derived for the external area for disturbances in the study area. This determination is carried out in two steps as shown in Figure 5.1. These are (i) coherency recognition and (ii) dynamic equivalencing and will be outlined in detail in the following sections.

5.2 COHERENCY RECOGNITION:

5.2.1 Conventional definition of coherency:

Traditionally, the term coherency has been defined with respect to machines that tend to swing together. Two generators 'i' and 'j' are said to be coherent [53] if there exists a constant α^*_{ij} such that

$$\delta_i(t) - \delta_j(t) = \alpha^*_{ij} \qquad (5.1)$$

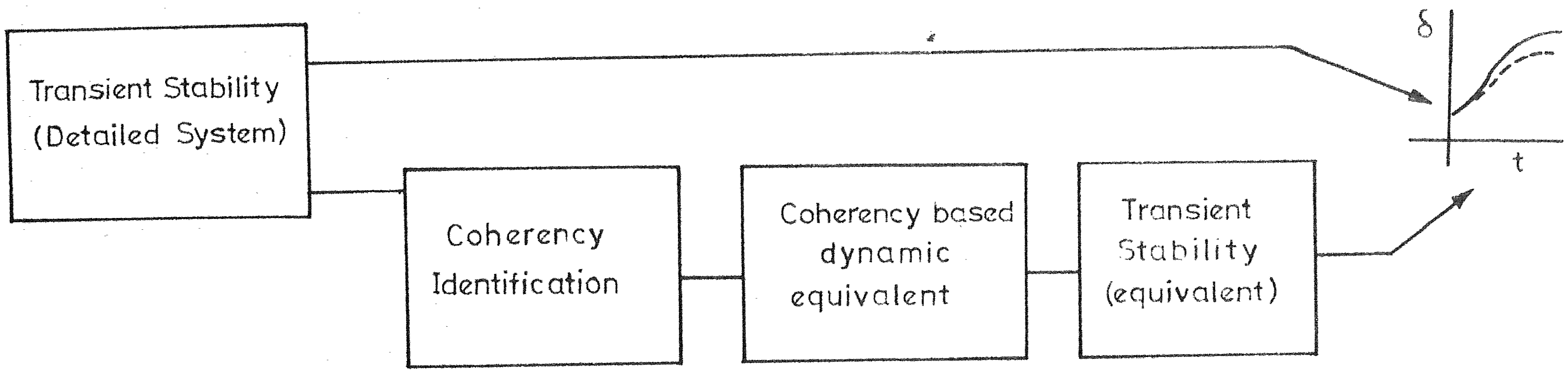

FIG. 5.1: MAIN STEPS IN THE DEVELOPMENT OF THE EQUIVALENT

for all time. Similarly a group of generators is said to be coherent if every pair of machines in the group is coherent. For example, consider a system consisting of 6-machines whose swing curves are as shown in Figure 5.2. It is evident that the angular difference δ_{12} between the machines 1 and 2 is constant subsequent to the disturbance at all time, i.e.,

$$\delta_{12}(t_o) = \delta_{12}(t_\infty) \qquad (5.2)$$

NOTE: In this chapter double subscript notation is introduced to denote the angular differences between two machines.

Similarly machines 3 and 5 maintain a constant angular difference. Thus two coherent groups with machines 1 and 2 in one and 3 and 5 in the other can be formed.

Differentiating (5.1) with respect to 't' and rearranging the terms one obtains

$$\omega_i(t) = \omega_j(t) = \omega(t) \quad \text{(say)}. \qquad (5.3)$$

Therefore if two or more machines are coherent, then their angular velocities are the same. The kinetic energies associated with the machines i and j are

$$\begin{aligned} K.E._i(t) &= \tfrac{1}{2} M_i \, \omega_i^2(t) \\ K.E._j(t) &= \tfrac{1}{2} M_j \, \omega_j^2(t) \end{aligned} \qquad (5.4)$$

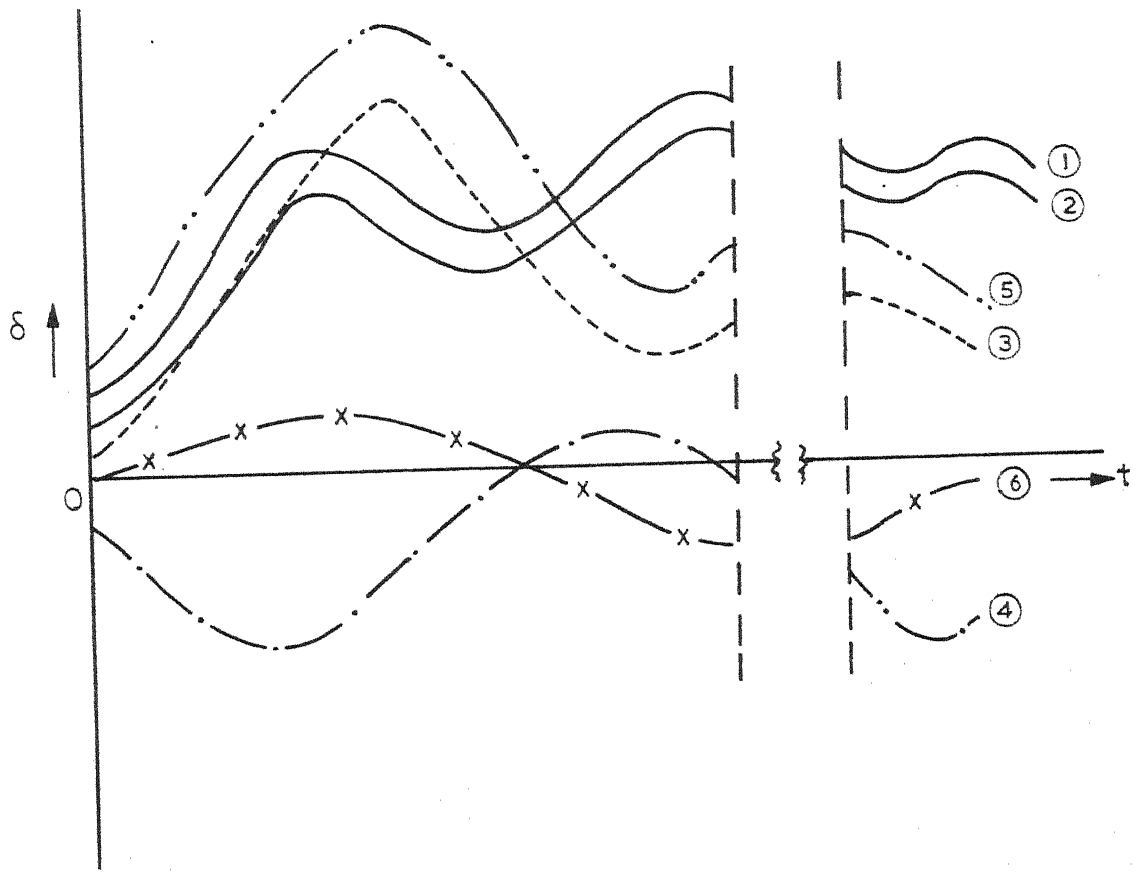

FIG.5.2: TYPICAL SWING CURVES OF A SIX MACHINE SYSTEM

Also the potential energy $P.E._{ij}(t)$ associated with this pair is given by

$$P.E._{ij}(t) = \int_0^{\sigma_{ij}} E_i E_j Y_{ij}[\cos(\theta_{ij}-\sigma_{ij}-\delta_{ij}^s) - \cos(\theta_{ij}-\delta_{ij}^s)]\, d\sigma_{ij} \qquad (5.5)$$

where $Y_{ij}\angle\theta_{ij} = G_{ij} + jB_{ij}$ is the short circuit transfer admittance between the machines i and j. δ_{ij}^s is the pre-fault stable angular difference between the two machines i and j. σ_{ij} is given by

$$\sigma_{ij} = \delta_{ij} - \delta_{ij}^s \quad . \qquad (5.6)$$

Under the assumption that all transfer conductances are neglected (5.5) reduces to

$$P.E._{ij}(t) = \int_0^{\sigma_{ij}} E_i E_j B_{ij}[\sin(\sigma_{ij}+\delta_{ij}^s)-\sin \delta_{ij}^s]\, d\sigma_{ij} \qquad (5.7)$$

The total energy $\mathscr{E}(t)$ associated with this pair of machines at any time 't' is then given by

$$\mathscr{E}(t) = \tfrac{1}{2} M_i\omega_i^2 + \tfrac{1}{2} M_j\omega_j^2 + \int_0^{\sigma_{ij}} E_i E_j B_{ij}[\sin(\sigma_{ij}+\delta_{ij}^s) - \sin \delta_{ij}^s]\, d\sigma_{ij} \quad . \qquad (5.8)$$

Equation (5.8) thus gives the total energy gained by this pair of machines i and j as the system evolves during a transient period from its pre-fault state. If however,

δ^s_{ij} is replaced by δ^o_{ij}, the post-fault steady state angular difference between the machines i and j we obtain

$$\mathcal{E}(t) = \tfrac{1}{2} M_i\omega_i^2 + \tfrac{1}{2} M_j\omega_j^2 + \int_0^{\sigma_{ij}} E_iE_j\hat{B}_{ij}[\sin(\sigma_{ij}+\delta^o_{ij}) - \sin \delta^o_{ij}]\, d\sigma_{ij} \; . \qquad (5.9)$$

where $\sigma_{ij} = \delta_{ij} - \delta^o_{ij}$ and $\hat{B}_{ij}$ is the transfer admittance between machines i and j in the post-fault state. This energy function in (5.9) has been extensively used as a Lyapunov function $V(\underline{X})$ in the stability analysis of power systems [5,6] and represents the energy gained by the system with the post-fault equilibrium as the datum. Either of the two equations (5.8) or (5.9) can be used for coherency recognition. In this chapter, however, equation (5.8) is used in what follows.

When the machines are coherent at all time, then

$$\delta_{ij}(t) = \delta^s_{ij} \quad \text{for all } t \geq t_o \qquad (5.10)$$

and will also be equal to δ^o_{ij} when a post-fault stable equilibrium state exists. Therefore the integral in (5.8) reduces to zero and the energy $\mathcal{E}(t)$ is given by

$$\mathcal{E}(t) = \tfrac{1}{2} M_i\omega_i^2(t) + \tfrac{1}{2} M_j\omega_j^2(t) \qquad (5.11)$$

and in view of (5.3),

$$\mathcal{E}(t) = \tfrac{1}{2}(M_i + M_j)\, \omega^2(t) \; . \qquad (5.12)$$

It is thus purely kinetic in nature. Hence the coherent

group is equivalent to a single machine whose inertia is the sum of the inertias of the individual generators. This is indeed the conventional method of obtaining the equivalent inertia of a single machine replacing a coherent group.

The fact that the integral vanishes provides an alternative criterion for identifying coherent machines which may be stated as follows:

5.2.2 An alternative criterion for coherency:

Two machines i and j are said to be coherent if the potential energy component associated with this pair of machines is zero for all time. Mathematically

$$E_i E_j B_{ij} \int_0^{\sigma_{ij}} [\sin(\sigma_{ij}+\delta^s_{ij}) - \sin \delta^s_{ij})] \, d\sigma_{ij} = 0 \quad (5.13)$$

Similarly a group of machines is said to be coherent if the potential energy component associated with every possible pair of machines in the group is zero. Thus for all possible angular differences $\delta_{ij}(t)$ for $t > t_o$, (5.13) is computed at every time step of integration of the differential equations and compared with a pre-assigned value.

During the computation it is possible to obtain extremely low values for the left hand side of (5.13) if the value of B_{ij} is extremely small. Such a situation occurs when the two machines under consideration are

electrically very far apart. To serve as a check whether or not the two machines are coherent, the slopes of the swing curves which yield the velocities are also monitored and compared. Instead of directly comparing the frequencies, the total kinetic energy is compared with the kinetic energy obtained by averaging the velocities, i.e. the expression

$$\tfrac{1}{2}\, M_i\omega_i^2(t) + \tfrac{1}{2}\, M_j\omega_j^2(t) - \tfrac{1}{2}(M_i+M_j)\,\omega^2(t) \qquad (5.14)$$

where $\omega(t) = \frac{1}{2}[\,\omega_i(t) + \omega_j(t)\,]$

is computed. This is equivalent to comparing the kinetic energy of the equivalent machine with the sum of the kinetic energies of the individual machines.

Perfect coherency in a practical system is very rare. It is therefore necessary to specify admissible tolerances on both the criteria (5.13) and (5.14) as follows:

$$P.E._{ij}(t) < \varepsilon_1 \qquad (5.15a)$$

$$[K.E._{ij}(t) - (K.E._i(t) + K.E._j(t)] < \varepsilon_2 \qquad (5.15b)$$

where ε_1 and ε_2 are the pre-assigned tolerances. Coherency identification using the criterion (5.15) is now possible as the stability study proceeds. At the end of every integration step the expression on the left side of equations (5.15) are computed for all possible pairs of machines. There will thus be $n(n-1)/2$ possible

computations to be carried out and those pairs that satisfy (5.15) are marked as coherent. From the sets so obtained, coherent groups are formed. As an example, consider a 5-machine system. If the criteria are satisfied by the pairs (2,3), (2,4) and (3,4) at a particular time step, then the coherent group of machines 2,3 and 4 is formed. At the end of the study, all groups that appear as coherent in every time step of integration are chosen. The procedure was implemented on a 15 machine, 44 bus and 56 line system and the results are summarised in Section 5.4.

5.3 DEVELOPMENT OF THE DYNAMIC EQUIVALENT:

In this section expressions for the admittance parameters in the dynamic equivalent obtained by replacing the coherent group of machines in the external area by an equivalent generator is obtained. Although results obtained for these parameters are identical to those of De Mello et al [56] the method of deriving the equivalent is new and utilizes the energy criteria developed in the previous section for coherency identification.

5.3.1 System description:

With the usual assumptions of (i) a constant voltage behind the transient reactance and (ii) all loads represented by constant impedances to ground, the self and transfer admittances between the generator internal nodes are first

derived. These are obtained by first augmenting an additional set of nodes to the bus admittance matrix of the system. These additional nodes represent the internal nodes of the generators. Then all the nodes excepting the generator internal nodes are eliminated. This results in the relationship

$$\underline{\bar{I}} = \bar{Y}\,\underline{\bar{E}} \tag{5.16}$$

where $\underline{\bar{I}}$ is a vector of complex current injections at the generator internal nodes and $\underline{\bar{E}}$ is a vector of complex internal voltages. A (-) over the variables represents complex quantities.

When the power system consists of groups of coherent generators it is possible to reduce the system model (5.16) by replacing each of these coherent groups by an equivalent generator. For example assume that in the 4-machine case of Figure 5.3(a) machines 3 and 4 form a coherent group. It will first be observed that a boundary between the coherent and the non-coherent areas through the nodes 3 and 4 can be drawn. This divides the system into two parts, one containing the non-coherent machines 1 and 2 and the other containing the coherent groups (3,4). The generator nodes 3 and 4 corresponding to the latter group lie on this boundary as shown in Figure 5.3(b) and can be replaced by

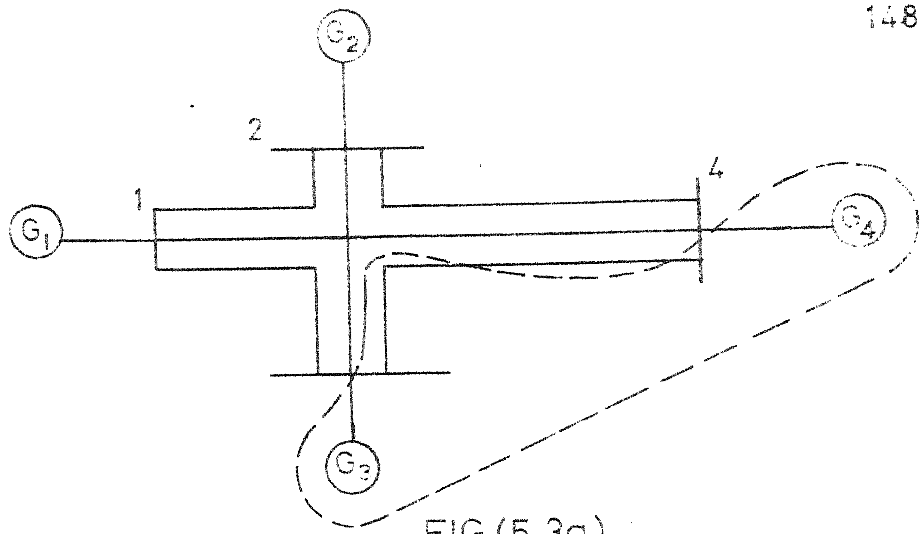

FIG (5.3a)

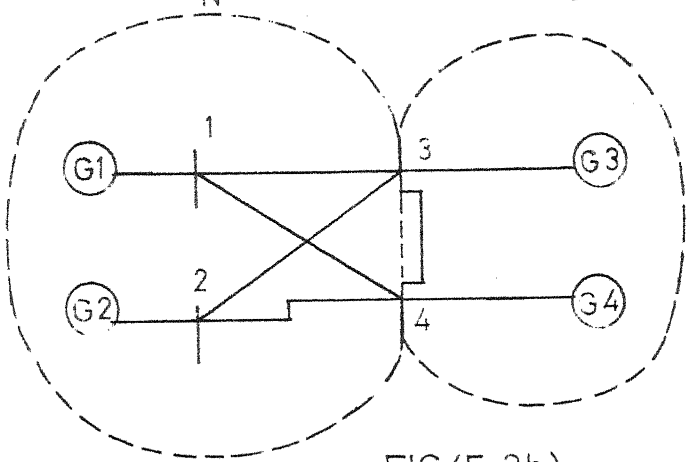

FIG (5.3b)

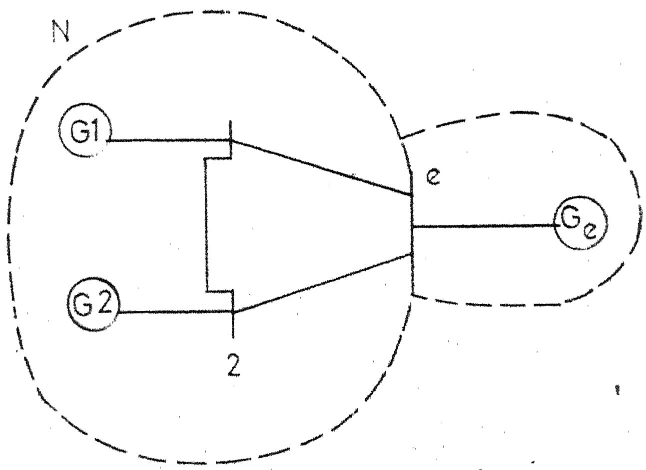

FIG (5.3c)

FIG. 5.3. SYSTEM DIVISION BASED ON COHERENCY

a single node 'e' and the corresponding generators by a single generator G_e as shown in Figure 5.3(c).

Assuming that only one coherent group of 'r' machines exists while the others swing independently it is possible to recast (5.16) in the form

$$\begin{bmatrix} \bar{I}_C \\ \bar{I}_N \end{bmatrix} = \begin{bmatrix} \bar{Y}_{CC} & \bar{Y}_{CN} \\ \bar{Y}_{NC} & \bar{Y}_{NN} \end{bmatrix} \begin{bmatrix} \bar{E}_C \\ \bar{E}_N \end{bmatrix} \tag{5.17}$$

where $\bar{I}_C$ = vector of complex current injections at the boundary or coherent internal nodes and of order r.

$\bar{I}_N$ = vector of complex current injections at the rest of the internal nodes and of the order m(= n-r)

$\bar{E}_C$ = vector of complex voltages at the boundary nodes

$\bar{E}_N$ = vector of complex voltages at the non-coherent nodes.

Subscripts C and N refer to the coherent and the non-coherent groups respectively. In the example of Figure 5.3(a) r corresponds to machines 3 and 4.

If the coherent nodes are to be replaced by an equivalent node e, the system representation will be of the form

$$\begin{bmatrix} \bar{I}_e \\ \bar{\underline{I}}_N \end{bmatrix} = \begin{bmatrix} \bar{Y}_{ee} & \bar{\underline{Y}}_{eN} \\ \bar{\underline{Y}}_{Ne} & \bar{Y}_{NN} \end{bmatrix} \begin{bmatrix} \bar{E}_e \\ \bar{\underline{E}}_N \end{bmatrix} \tag{5.18}$$

where $\bar{I}_e$ = current injection at the equivalent node'e'

$\bar{E}_e$ = voltage at the node 'e'.

Comparing (5.17) and (5.18) it may be noted that the terms in $\bar{Y}_{NN}$ are unchanged. Explicit expressions for $\bar{Y}_{ee}$ and $\bar{\underline{Y}}_{Ne}$ will now be obtained. As will be shown later, $\bar{\underline{Y}}_{eN}$ is made equal to $\bar{\underline{Y}}_{Ne}^T$.

5.3.2 Computation of $\bar{Y}_{ee}$ and $\bar{\underline{Y}}_{eN}$:

Expanding (5.17) one obtains

$$\begin{bmatrix} \bar{I}_{C1} \\ \bar{I}_{C2} \\ \cdot \\ \bar{I}_{Cr} \\ \hline \bar{I}_{N1} \\ \bar{I}_{N2} \\ \cdot \\ \bar{I}_{Nm} \end{bmatrix} = \left[\begin{array}{cccc|cccc} \bar{Y}_{C1,C1} & \bar{Y}_{C1,C2} & \cdots & \bar{Y}_{C1,Cr} & \bar{Y}_{C1,N1} & \cdots & \bar{Y}_{C1,Nm} \\ \bar{Y}_{C2,C1} & \bar{Y}_{C2,C2} & \cdots & \bar{Y}_{C2,Cr} & \bar{Y}_{C2,N1} & \cdots & \bar{Y}_{C2,Nm} \\ \cdot & \cdot & \cdots & \cdot & \cdot & \cdots & \cdot \\ \bar{Y}_{Cr,C1} & \bar{Y}_{Cr,C2} & \cdots & \bar{Y}_{Cr,Cr} & \bar{Y}_{Cr,N1} & \cdots & \bar{Y}_{Cr,Nm} \\ \hline \bar{Y}_{N1,C1} & \bar{Y}_{N1,C2} & \cdots & \bar{Y}_{N1,Cr} & \bar{Y}_{N1,N1} & \cdots & \bar{Y}_{N1,Nm} \\ \bar{Y}_{N2,C1} & \bar{Y}_{N2,C2} & \cdots & \bar{Y}_{N2,Cr} & \bar{Y}_{N2,N1} & \cdots & \bar{Y}_{N2,Nm} \\ \cdot & \cdot & \cdots & \cdot & \cdot & \cdots & \cdot \\ \bar{Y}_{Nm,C1} & \bar{Y}_{Nm,C2} & \cdots & \bar{Y}_{Nm,Cr} & \bar{Y}_{Nm,N1} & \cdots & \bar{Y}_{Nm,Nm} \end{array}\right] \begin{bmatrix} \bar{E}_{C1} \\ \bar{E}_{C2} \\ \cdot \\ \bar{E}_{Cr} \\ \hline \bar{E}_{N1} \\ \bar{E}_{N2} \\ \cdot \\ \bar{E}_{Nm} \end{bmatrix} \tag{5.19}$$

Subscripted numbers in (5.19) denote the nodes in the respective groups N or C. The injected currents $\bar{I}_{Ci}$ at

any ith machine in group C and at any ℓth node in group N are respectively given by the following equations.

$$\bar{I}_{Ci} = \bar{Y}_{Ci,C1}\,\bar{E}_{C1} + \bar{Y}_{Ci,C2}\,\bar{E}_{C2} + \cdots + \bar{Y}_{Ci,Ci}\,\bar{E}_{Cr} +$$

$$+ \bar{Y}_{Ci,N1}\,\bar{E}_{N1} + \cdots + \bar{Y}_{Ci,Nm}\,\bar{E}_{Nm}$$

$$= \sum_{j=1}^{r} \bar{Y}_{Ci,Cj}\,\bar{E}_{Cj} + \sum_{k=1}^{m} \bar{Y}_{Ci,Nk}\,\bar{E}_{Nk} \tag{5.20}$$

and

$$\bar{I}_{N\ell} = \sum_{j=1}^{r} \bar{Y}_{N\ell,Cj}\,\bar{E}_{Cj} + \sum_{k=1}^{m} \bar{Y}_{N\ell,Nk}\,\bar{E}_{Nk}\,. \tag{5.21}$$

From equation (5.18) for the reduced system the current injection at the eth node is given by

$$\bar{I}_{e} = \bar{Y}_{ee}\,\bar{E}_{e} + \sum_{k=1}^{m} \bar{Y}_{e,Nk}\,\bar{E}_{Nk} \tag{5.22}$$

and in the group N, at the ℓth node

$$\bar{I}_{N\ell} = \bar{Y}_{N\ell}\,\bar{E}_{e} + \sum_{k=1}^{m} \bar{Y}_{N\ell,Nk}\,\bar{E}_{Nk}\,. \tag{5.23}$$

We now seek to develop expressions for $\bar{Y}_{ee}$ and $\underline{\bar{Y}}_{Ne}$ utilizing the criterion developed in Section 5.2.2 for coherency which states that the system energy in the reduced and the unreduced system be the same. The total energy $\mathcal{E}(t)$ of the unreduced system at any time t with transfer conductances neglected is given by

$$\mathcal{E}(t) = \sum_{i=1}^{m+r} \tfrac{1}{2} M_i\omega_i^2 + \sum_{i=1}^{m+r-1} \sum_{j=i+1}^{m+r} \int_0^{\sigma_{ij}} E_iE_jB_{ij}[\sin(\sigma_{ij}+\delta^s_{ij})$$

$$- \sin \delta^s_{ij}]d\sigma_{ij} \ . \qquad (5.24)$$

(5.24) may be written as

$$\mathcal{E}(t) = \tfrac{1}{2} \sum_{\substack{i=1 \\ i\varepsilon N}}^{m} M_i\omega_i^2(t) + \tfrac{1}{2} \sum_{\substack{k=1 \\ k\varepsilon C}}^{r} M_k\omega_k^2$$

$$+ \sum_{\substack{i=1 \\ i\varepsilon C}}^{r-1} \sum_{\substack{j=i+1 \\ j\varepsilon C}}^{r} \int_0^{\sigma_{ij}} E_iE_jB_{ij}[\sin(\sigma_{ij}+\delta^s_{ij})-\sin\delta^s_{ij}]d\sigma_{ij}$$

$$+ \sum_{\substack{k=1 \\ k\varepsilon N}}^{m-1} \sum_{\substack{\ell=k+1 \\ \ell\varepsilon N}}^{m} \int_0^{\sigma_{k\ell}} E_kE_\ell B_{k\ell}[\sin(\sigma_{k\ell}+\delta^s_{k\ell})-\sin\delta^s_{k\ell}]d\sigma_{k\ell}$$

$$+ \sum_{\substack{p=1 \\ p\varepsilon C}}^{r} \sum_{\substack{q=1 \\ q\varepsilon N}}^{m} \int_0^{\sigma_{ij}} E_pE_qB_{pq}[\sin(\sigma_{pq}+\delta^s_{pq})-\sin\delta^s_{pq}]d\sigma_{pq} \qquad (5.25)$$

It may be noted that the subscripts N and C for the voltages, denoting the groups to which these voltages belong as shown in (5.19) are omitted for convenience. The first two summation terms in (5.25) represent the kinetic energies of the machines in the groups N and C. The third term is the potential energy in the coherent group C and is therefore zero. The fourth is similar to the third but pertains to group N but is not zero. The last term which is of interest represents the potential energy associated with every pair of machines obtained by choosing one from C and the other from N.

In a similar manner for the equivalent, the energy $\mathcal{E}^*(t)$ is given by

$$\mathcal{E}^*(t) = \tfrac{1}{2} \sum_{\substack{i=1 \\ i\varepsilon N}}^{m} M_i \omega_i^2(t) + \tfrac{1}{2} M_e \omega^2(t)$$

$$+ \sum_{\substack{k=1 \\ k\varepsilon N}}^{m-1} \sum_{\substack{\ell=k+1 \\ \ell\varepsilon N}}^{m} \int_0^{\sigma_{k\ell}} E_k E_\ell B_{k\ell} [\sin(\sigma_{k\ell} + \delta_{k\ell}^s) - \sin\delta_{k\ell}^s] d\sigma_{k\ell}$$

$$+ \sum_{\substack{q=1 \\ q\varepsilon N}}^{m} \int_0^{\sigma_{eq}} E_e E_q B_{eq} [\sin(\sigma_{eq} - \delta_{eq}^s) - \sin\delta_{eq}^s] d\sigma_{eq} \qquad (5.26)$$

The first and the third terms in (5.26) correspond to machines in N and are the same as the first and the fourth terms respectively in (5.25). We note that the coherent group can be replaced by an equivalent machine of inertia M_e given by

$$M_e = \sum_{\substack{i=1 \\ i\varepsilon C}}^{r} M_i \qquad (5.27)$$

and also $\omega_k(t)$, $(k\varepsilon C) = \omega(t)$.

Now equating the energies $\mathcal{E}^*(t)$ to $\mathcal{E}(t)$ we get

$$\sum_{\substack{q=1 \\ q\varepsilon N}}^{m} \int_0^{\sigma_{eq}} E_e E_q B_{eq} [\sin(\sigma_{eq} - \delta_{eq}^s) - \sin\delta_{eq}^s] d\sigma_{eq}$$

$$= \sum_{\substack{p=1 \\ p\varepsilon C}}^{r} \sum_{\substack{q=1 \\ q\varepsilon N}}^{m} \int_0^{\sigma_{pq}} E_p E_q B_{pq} [\sin(\sigma_{pq} + \delta_{pq}^s) - \sin\delta_{pq}^s] d\sigma_{eq} \qquad (5.28)$$

For any arbitrary q in N, equation (5.28) yields

$$\int_0^{\sigma_{eq}} E_e E_q B_{eq}[\sin(\sigma_{eq}+\delta_{eq}^s)-\sin\delta_{eq}^s]d\sigma_{eq}$$

$$= \sum_{\substack{p=1 \\ p\varepsilon C}}^{r} \int_0^{\sigma_{pq}} E_p E_q B_{pq}[\sin(\sigma_{pq}+\delta_{pq}^s)(-\sin\delta_{pq}^s]d\sigma_{pq} \ . \qquad (5.29)$$

E_e is not known a priori. It is arbitrary and is chosen as some sort of an average of the voltage at the buses in the coherent group. Different methods of selecting $\bar{E}_e$ have been proposed in reference [56]. In this analysis, the following choice of $\bar{E}_e$ overcomes the difficulty in solving for B_{eq} from (5.29):

$$\bar{E}_e = E_e \angle\delta_e^s = \sqrt[r]{\prod_{p=1}^{r} E_p} \Big/ \frac{1}{r}\sum_{p=1}^{r} \delta_p^s \ . \qquad (5.30)$$

Equation (5.30) implies that $\bar{E}_e$ is the geometric mean of the complex voltages of the coherent nodes. With this choice it is trivial to show that

$$\sigma_{pq} = \sigma_{eq}, \ p = 1,2,\ldots r. \qquad (5.31)$$

Hence equating the integrands on either sides of (5.29) one obtains

$$E_e E_q B_{eq}[\sin(\sigma_{eq}+\delta_{eq}^s)-\sin\delta_{eq}^s]$$

$$= \sum_{\substack{p=1 \\ p\varepsilon C}}^{r} E_p E_q B_{pq}[\sin(\sigma_{pq}+\delta_{pq}^s)-\sin\delta_{pq}^s] \qquad (5.32)$$

Since $\delta_{pq} - \delta_{pq}^s = \sigma_{pq}$ and $\delta_{eq}-\delta_{eq}^s = \sigma_{eq}$, one obtains

from (5.32)

$$E_e\ E_q\ B_{eq}\ \sin\ \delta_{eq} - E_e\ E_q\ B_{eq}\ \sin\ \delta^s_{eq}$$

$$= \sum_{\substack{p=1\\ p\varepsilon C}}^{r} E_p\ E_q\ B_{pq}\ \sin\ \delta_{pq} - \sum_{\substack{p=1\\ p\varepsilon C}}^{r} E_p\ E_q\ B_{pq}\ \sin\ \delta^s_{pq}\ . \quad (5.33)$$

The first term on the left side of (5.33) represents the power transfer from the equivalent bus to the node'q'in N and the second term represents the power transfer at the initial time t_o. Similarly the first and second terms on the right side of (5.33) yields the power transfer from the coherent group to node 'q' at 't' and 't_o' respectively. Since energy transfer is to be same at all time, the power transfer at all times should also be equal. Hence the first terms on either sides can be equated to give

$$E_e E_q B_{eq}\ \sin\ \delta_{eq} = \sum_{\substack{p=1\\ p\varepsilon C}}^{r} E_p\ E_q\ B_{pq}\ \sin\ \delta_{pq}$$

$$\text{or}\quad \mathrm{Re}[\bar{E}_q\ \bar{E}^*_e\ \bar{Y}^*_{qe}] = \sum_{\substack{p=1\\ p\varepsilon C}}^{r} \mathrm{Re}[\bar{E}_q\ \bar{E}^*_p\ \ \bar{Y}^*_{qp}] \quad (5.34)$$

NOTE: * indicates conjugation.

Therefore,

$$\bar{E}^*_q\ \bar{E}_e\ \bar{Y}_{qe} = \sum_{\substack{p=1\\ p\varepsilon C}}^{r} \bar{E}^*_q\ \bar{E}_p\ \bar{Y}_{qp}$$

from which

$$\bar{Y}_{qe} = \sum_{p=1}^{r} (\bar{E}_p/\bar{E}_e)\ \bar{Y}_{pq} \tag{5.35}$$

Thus the qth element of $\underline{\bar{Y}}_{Ne}$ is obtained. This result agrees with that obtained by Bree etal [56]. It now remains to obtain $\bar{Y}_{ee}$.

To compute $\bar{Y}_{ee}$ it will be assumed that the net energy fed into the unreduced system by the coherent group is exactly equal to the energy fed into the reduced system by the equivalent machine at all time. This implies that the net power injection 'P_e' at the equivalent node 'e' in the dynamic equivalent equals the sum of the net injections P_i, i = 1,2,...r at the internal nodes of the coherent machines in the unreduced system.

The procedure therefore becomes identical to that by Bree etal [56].

The power P_i injected at any node 'i' in C is given by

$$P_i = \bar{E}_{Ci}\ \bar{I}^*_{Ci}\ . \tag{5.36}$$

The total power P_c injected into the system by C is

$$P_c = \sum_{i=1}^{r} \bar{I}^*_{Ci}\ \bar{E}_{Ci}\ . \tag{5.37}$$

and the power injection P_e at the equivalent node is

$$P_e = \bar{I}_e^*\ \bar{E}_e\ . \tag{5.38}$$

Substituting (5.20) in (5.37) one obtains

$$P_c = \sum_{j=1}^{r} \sum_{i=1}^{r} \bar{E}^*_{Cj}\ \bar{Y}^*_{Ci,Cj}\ \ \bar{E}_{Ci} + \sum_{k=1}^{m} \sum_{i=1}^{r} \bar{E}^*_{Nk}\ \ \bar{Y}^*_{Ci,Nk}\ \bar{E}_{Ci} \tag{5.39}$$

Similarly using (5.22) in (5.38), P_e is given by

$$P_e = \bar{E}_e^* \bar{Y}_{ee}^* \bar{E}_e + \sum_{k=1}^{m} \bar{E}_{Nk}^* \bar{Y}_{e,Nk}^* \bar{E}_e \tag{5.40}$$

The second terms in (5.39) and (5.40) represent the net power flow into the system through the nodes of group N and the equivalent respectively while the first terms give the net powers used up at the nodes of C and the equivalent respectively. Since the transfer admittance between nodes 'e' and 'Nk' in the equivalent is known from (5.35), by equating P_e and P_c and simplifying $\bar{Y}_{ee}$ is obtained as

$$\bar{Y}_{ee} = \frac{1}{E_e^2} \sum_{i=1}^{r} \sum_{j=1}^{r} \bar{E}_{Ci}^* \bar{Y}_{Ci,Cj}^* \bar{E}_{Cj} + \frac{1}{E_e^2} \sum_{k=1}^{m} \sum_{i=1}^{r} \left(\bar{E}_{Ci}^* \bar{Y}_{Ci,Nk}^* \bar{E}_{Nk} - \frac{\bar{E}_e^*}{\bar{E}_e} \bar{E}_{Ci} \bar{Y}_{Ci,Nk} \bar{E}_{nk}\right). \tag{5.41}$$

Thus (5.35) and (5.41) yield values of the unknown parameters in the admittance matrix of (5.18). The elements in Y_{NN} of (5.17) and (5.18) are unaffected. From these equations it is clear that for any coherent group, one row of elements has to be evaluated corresponding to node 'e' in the equivalent and involves only multiplications, while the dynamic equivalent developed in reference [53] using the node elimination formula involves

matrix inversions. Considerable reduction in time is thus effected in the present method of equivalencing.

5.4 NUMERICAL EXAMPLE:

The test system chosen for study is the 44 bus, 15 machine and 56 lines system shown in Figure 5.4. The data for this system is given in Appendix E. The region encompassing the buses 1,2,3,4 and 6 was assumed to be the study area. Buses 1 and 2 are generator buses while the rest are non-generator buses. The performance of the machines 1 and 2 in the study area was analyzed when a fault on bus 6 was cleared after 0.1 second.

In order to identify coherent machines, a detailed stability study was carried out on the unreduced system with the generator represented by a constant voltage behind their respective transient reactances. Since the purpose of this study is (i) to demonstrate the effectiveness of the coherency criterion developed in Section 5.2 and (ii) the feasibility of the coherency based dynamic equivalent derived in Section 5.3, to large scale power systems the computations of the quantities on the left hand sides of equations (5.14) for coherency recognition at the end of every time step was included in the program. The output lists out pairs of generators that are coherent according to the tolerences specified. Various values of ε_1 and ε_2

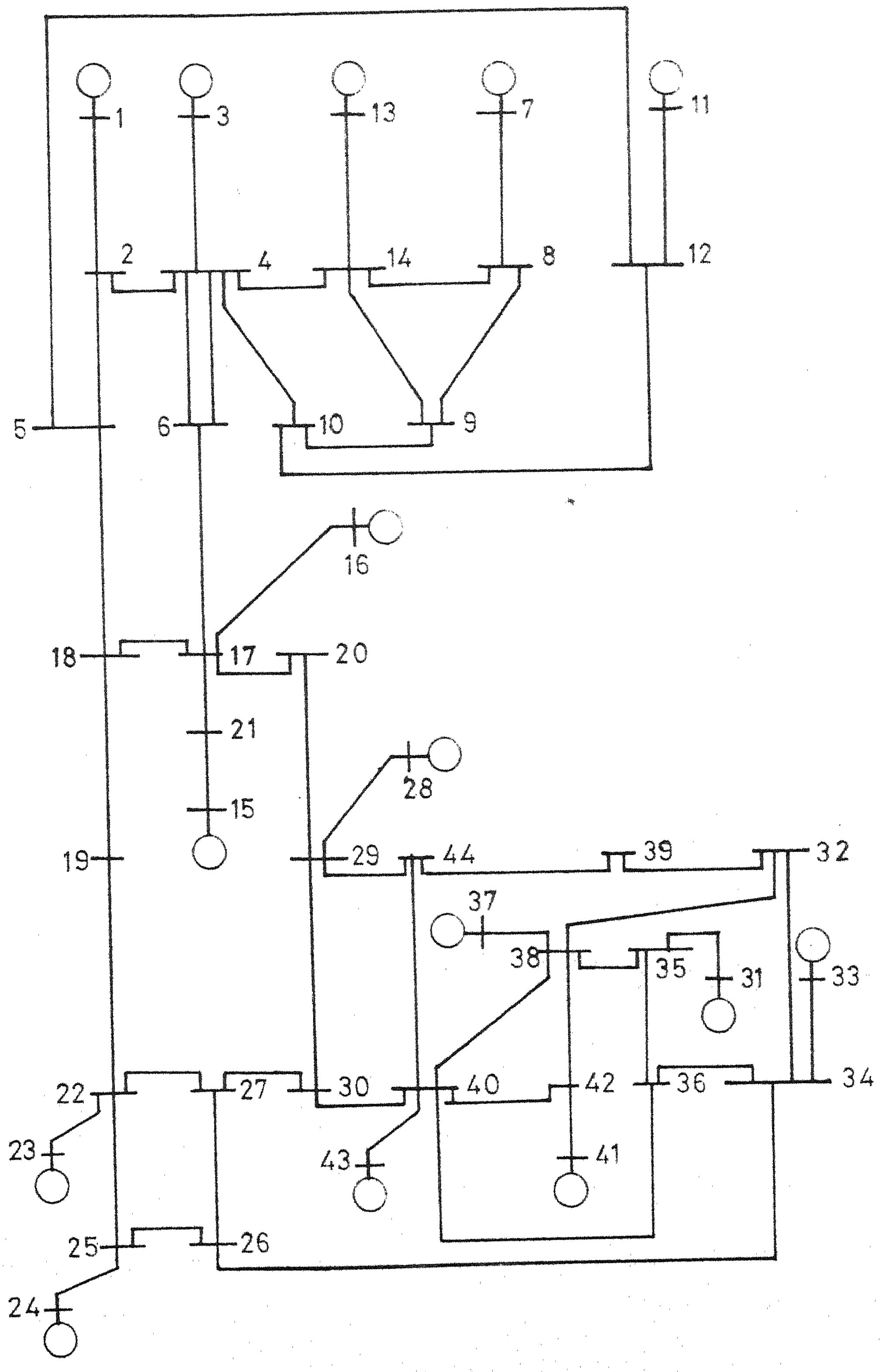

FIG. 5.4: 44 BUS SAMPLE SYSTEM

were tried and their effect on coherency was investigated. Table 5.1 gives the coherent groups for different tolerances assigned to ε_1 and ε_2. With a value of 0.0001 assigned to each, only one group consisting of machines 13,14,15 in the external system was found as coherent. This implies that these machines are tightly coupled. Swing curves plotted for these machines in Figure 5.5 show perfect coherency between them. A relaxation in ε_1 or ε_2 yields another coherent group comprising machines 9 and 12. Swing curves for these machines are shown in Figure 5.6. A rough visual inspection of the swing curves of these machine groups (13,14,15) and (9,12) would indicate that all these machines are coherent. However, this is critically dependent upon the values of ε_1 and ε_2 (cases 2 and 3 in Table 5.1). With ε_1 and ε_2 as above two distinct groups are indicated. Loosening the restriction by allowing ε_1 and $\varepsilon_2 \doteq 0.0005$ makes all 5 machines coherent. This study thus reveals the importance of a proper choice of ε_1 and ε_2 for detecting coherency.

Having obtained the coherent groups, the data pertaining to these machines is fed to the equivalencing program that yields the dynamic equivalent of the system. The program computes the voltages, inertias, power input and self and transfer admittances of the equivalent generator replacing the coherent group. For this, the two groups obtained in case 2 of Table 5.1 were used.

Table 5.1: Effect of ε_1 and ε_2 on coherency.

Case No.	ε_1	ε_2	MACHINES IN COHERENT GROUP NO. 1	2
1	0.0001	0.0001	13,14,15	-
2	0.0001	0.0005	13,14,15	9,12
3	0.0005	0.0001	13,14,15	9,12
4	0.0005	0.0005	9,12,13,14,15	-

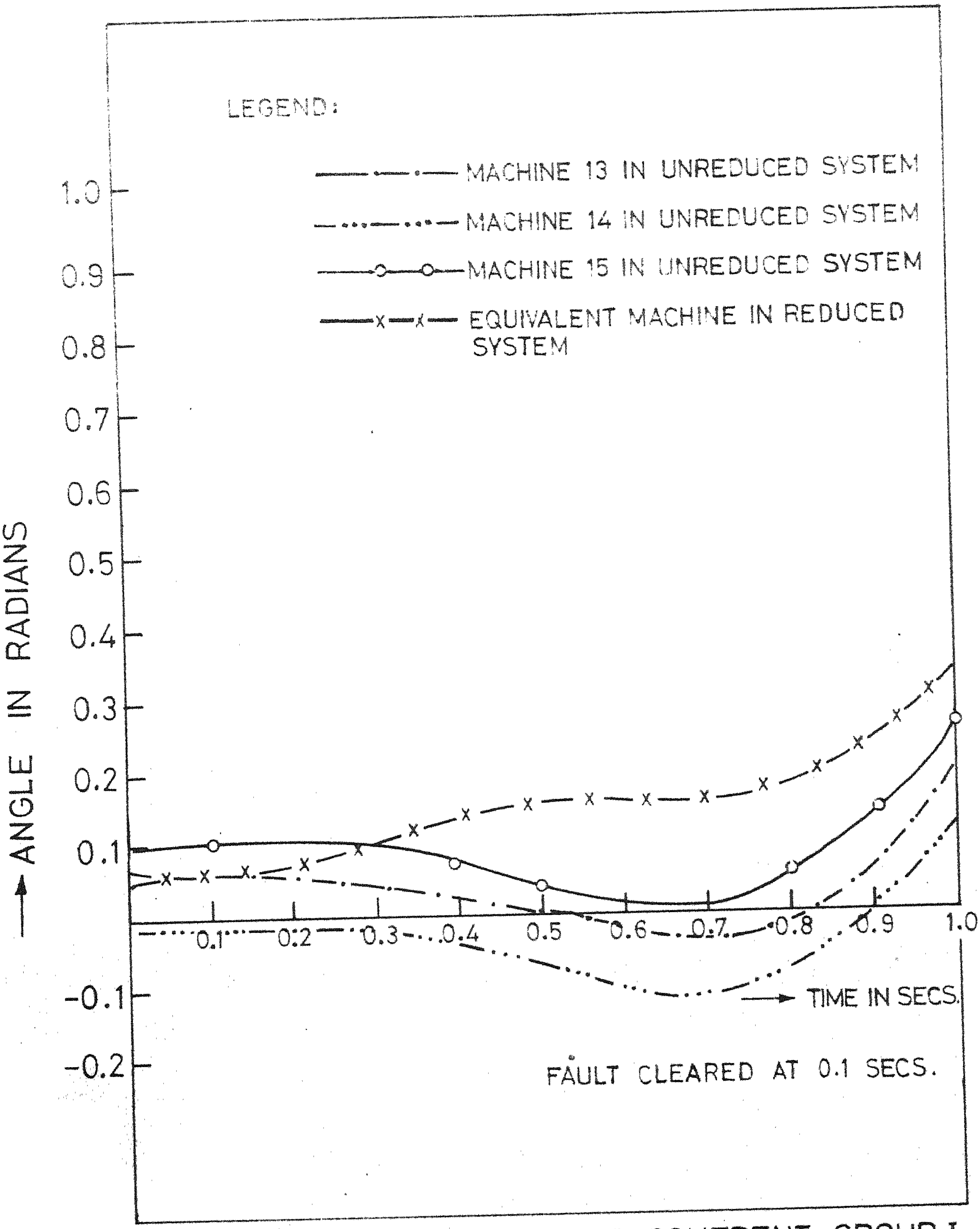

FIG. 5.5 SWING CURVES OF COHERENT GROUP I AND THEIR EQUIVALENT

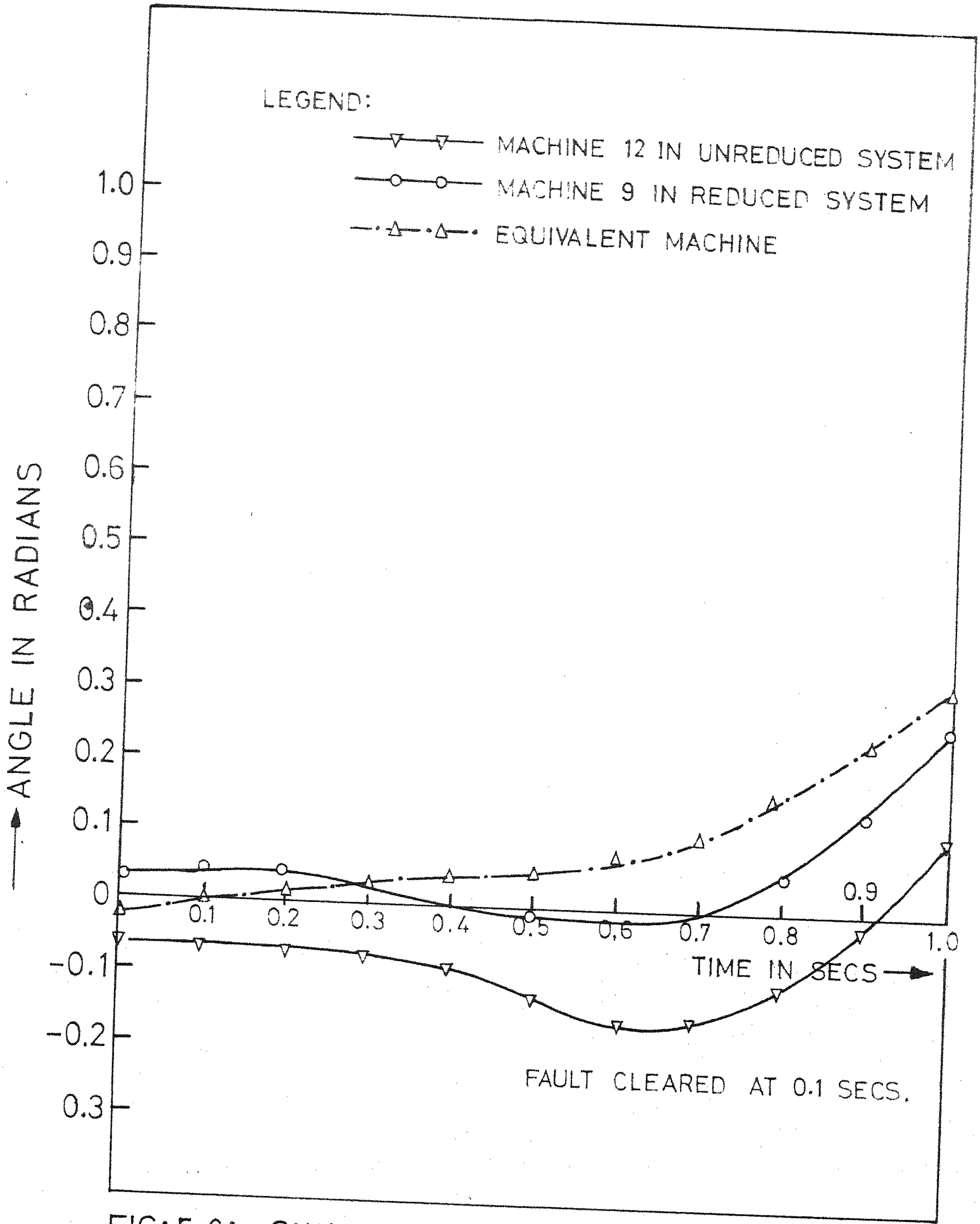

FIG: 5.6: SWING CURVES OF COHERCNT GROUP II AND THEIR EQUIVALENT

A stability study was again conducted with these coherent groups replaced by their equivalent in order to obtain the behaviour of machines 1 and 2 in the study area. Figures 5.7 and 5.8 show the swing curves of these two machines obtained from the detailed study and using the equivalents. Close agreement is observed. The swing curves of the equivalent machines replacing the coherent groups are shown in Figures 5.5 and 5.6. These curves indicate that the equivalent reacts faster than the machines it replaces. This is due to the fact that the electrical distance of this equivalent generator from the disturbance is reduced. But the nature of the swing is similar to the machines in the corresponding coherent group. Since the behaviour of the machines 1 and 2 in the study area is of concern, the equivalent representing the machines in the external area can be considered to have behaved well. The swing curves of machine '5' retained in the external area is also shown in Figure 5.9 obtained from a stability study on the reduced and unreduced system. Good agreement is observed.

5.5 CONCLUSIONS:

The necessity of an accurate equivalent that simulates the behaviour of the original system for stability studies has been felt in the recent years. An effective method to detect coherency in the system based

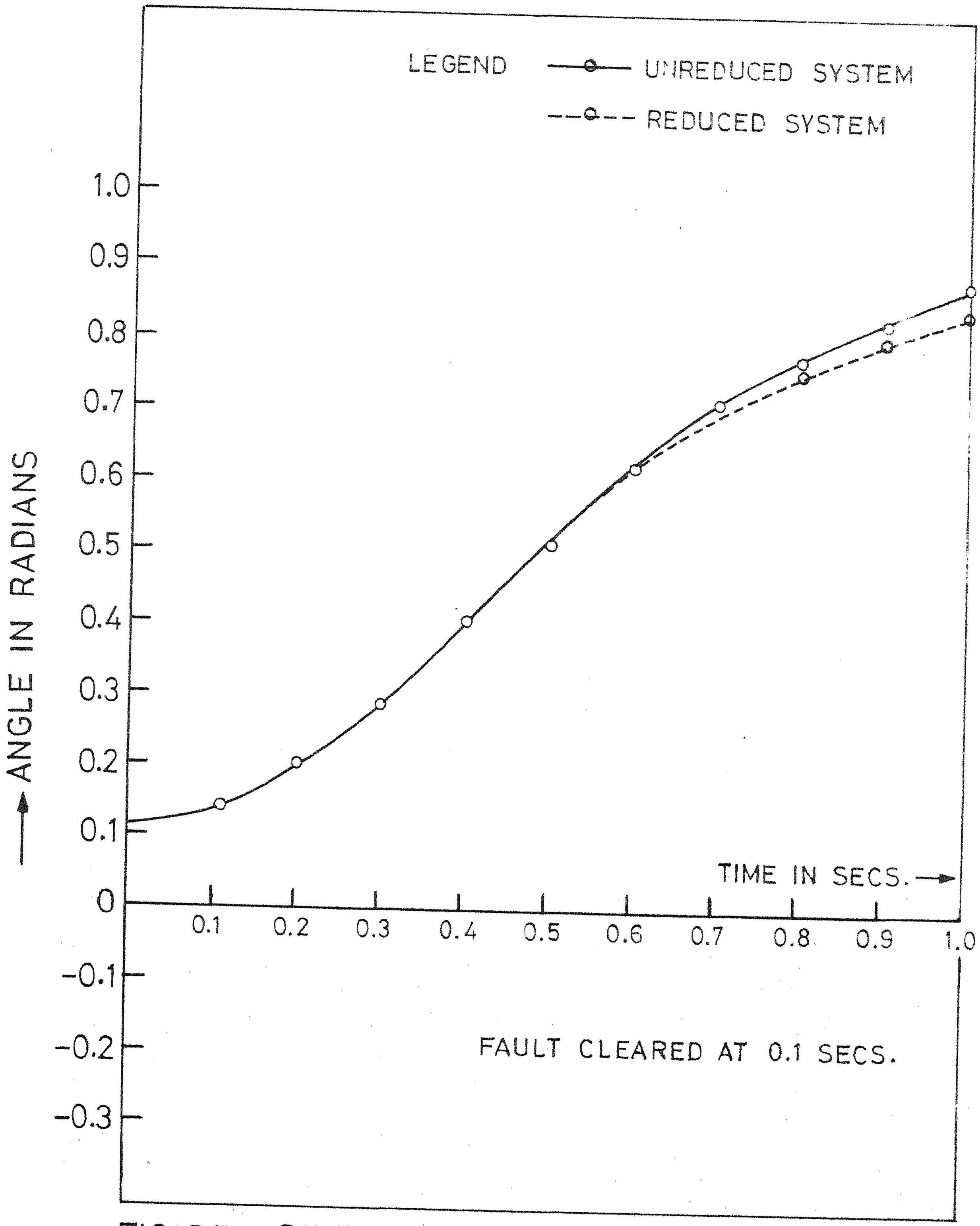

FIG. 5.7: SWING CURVES OF MACHINE 1
(STUDY AREA)

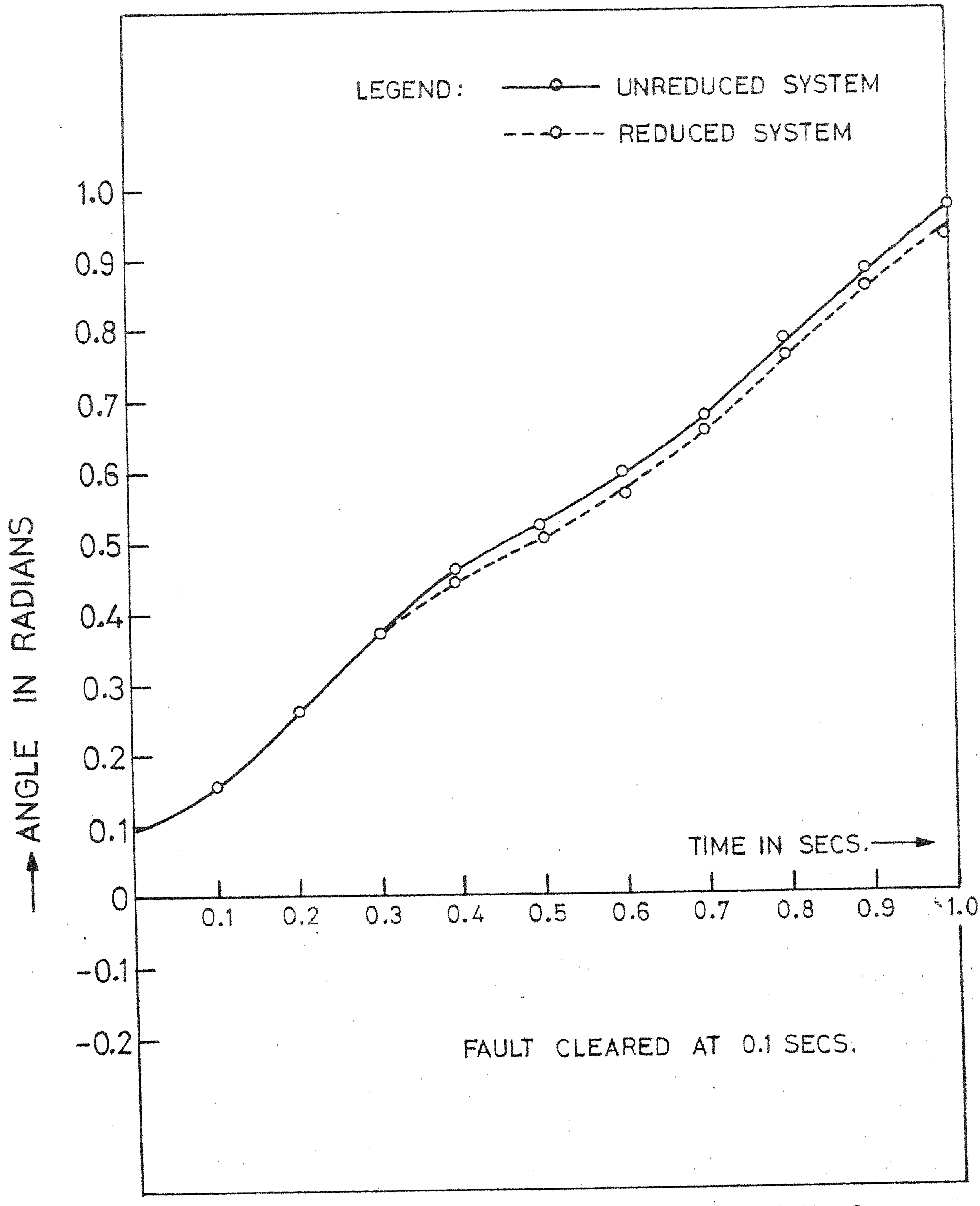

FIG. 5.8: SWING CURVES OF MACHINE 2 (STUDY AREA)

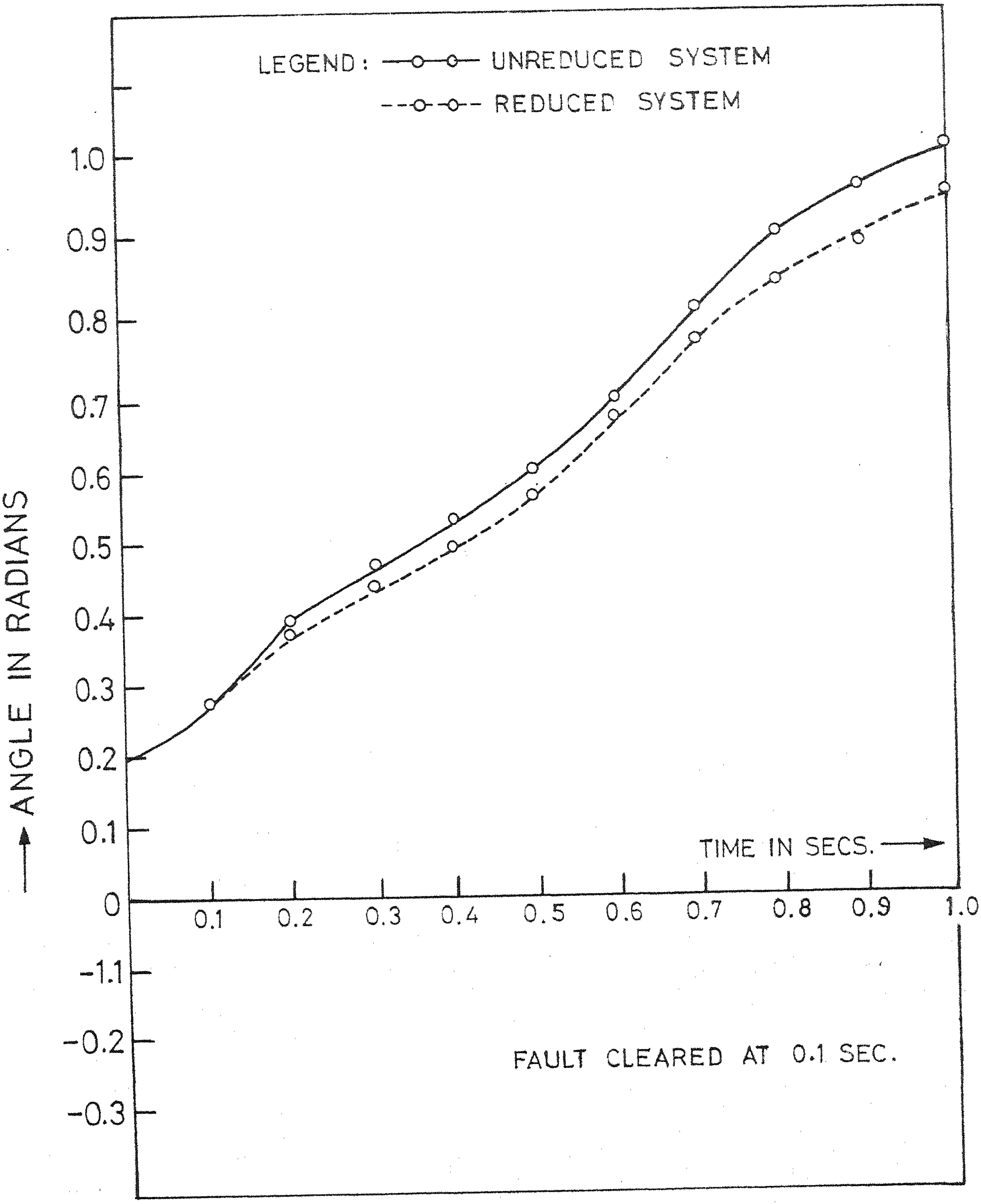

FIG. 5.9: SWING CURVES OF MACHINE 5

on energetical considerations has been presented in this chapter. The responses of the machines in a coherent group obtained by this technique, were found to be perfectly matching as is evident from the swing curves. The dynamic equivalent derived utilizing the property of coherency is found to yield responses for machines in the study area, identical to those from a detailed simulation. Such an equivalent is extremely useful in the stability analysis of large scale power systems using Lyapunov methods since the number of unstable equilibrium points and hence their computation is greatly reduced.

CHAPTER VI

CONCLUSIONS

6.1 SUMMARY AND CONCLUSIONS

The development of Lyapunov theory for the stability analysis of power systems has been carried out with sufficient interest in recent years. The method has proven to be an attractive alternative to the simulation techniques both from the theoretical and practical points of view. While sufficient work has been carried out for the construction of Lyapunov functions, the application of the method to realistic power systems from the practical point of view has been slow. Apart from the general conservative nature of the method inherent in the Lyapunov theorem itself, there are still certain drawbacks still to be overcome. One of these drawbacks is in regard to multimachine system modelling. It has been difficult to include effects like saturation flux decay, governor controls and transfer conductances in constructing a Lyapunov function. Hence the models were restricted to the simplest type. Therefore, alternate approaches are called for in order to consider these factors in the construction of a suitable Lyapunov function. Some of these include the use of vector Lyapunov functions and the use of simplified models called the dynamic equivalents.

Another drawback of the Lyapunov method for the stability analysis of power systems has been in regard to the determination of the stability boundary. The problem involves the computation of a large number of unstable equilibrium points and demands a considerable amount of computer time. This difficulty has been obviated to a large extent in a recent work of Prabhakara and El-Abiad. Improvements on this method in obtaining better stability boundaries are possible.

In this research, emphasis has been, therefore, given to some aspects of the above two problems. The following are some of the conclusions arrived at.

The nature of power system state models for the construction of Lyapunov functions has been a topic of considerable controversy over the past few years. It has been agreed that some of these models are uncontrollable and unobservable and hence cannot be used for the construction of Lyapunov functions using systematic procedures of Kalman or the Moore-Anderson theorem. Although this controversy is by now settled via the control theoretic concepts of minimal realization, reasons for uncontrollability in some of the models have not fully been explained. It has been shown in Chapter II that there is a physical dependency existing among the state variables in such models resulting in overdescription

and this indeed seems to be the cause of the so-called uncontrollability of some of the models used in the literature.

The new approach of vector Lyapunov function introduced in Chapter III is a two level concept and utilizes the decomposition and aggregation techniques for the stability analysis of large scale power systems. Decomposition of the system is a key step in the application of these concepts. A procedure for decomposing the power system into low order subsystems has been proposed. The nature of decomposition is such that each of these consists of two machine groups. It is therefore possible to include the system refinements in each of these subsystems at the lower level and is a topic of further investigation. For the simple model considered at the subsystem level, Lyapunov functions have been constructed. Using the properties of these Lyapunov functions and the nature of interactions among the subsystems, the application of vector Lyapunov function at the higher hierarchiel level has been demonstrated. Preliminary results indicate that the method yields more conservative stability regions. However, the possibility of including system refinements outweighs this disadvantage. Also recent advances in large scale system stability theory indicate that sharper results may be obtained.

The subject matter of Chapter IV has dealt with the determination of the stability boundary of a power system by a method that avoids the computation of the unstable equilibrium points. The use of the sector violation information of the nonlinearities in the system model (which is in the Lure'-Popov form) in obtaining the stability region for a multinonlinear system has been discussed. The results obtained are shown to be a generalization of the work of a similar nature in connection with the estimation of stability domain for a system with a single nonlinearity. On the basis of these results, the problem of estimating the stability region for the multi-machine power system is analyzed. Numerical computations on the 5-machine example presented and on other systems [79] indicate that the method yields conservative estimates of the domains. But the simplicity of computation is more attractive.

Chapter V has presented a new method of identifying coherency and developing dynamic equivalents using energy functions. It frequently happens that certain section say 'A' of the system is not adversely affected by a disturbance in another section 'B'. In such a case some of the machines in 'A' tend to swing almost together during the period under study in groups. Such groups of coherent machines can be replaced by a

single equivalent machines for each group. A reduced order dynamic equivalent of the system 'A' can then be derived. In this chapter, coherency identification and the development of the equivalent are based on certain energy criteria. For this purpose an energy type Lypunov function is used. An advantage of the method is that the angular differences at any time during the period of study is always compared with those of the initial angular differences. Results of tests conducted on the 44 bus system show that the method is extremely effective in respect of coherency recognition. The dynamic equivalent produces swing curves for the retained machines in almost full agreement with those obtained by a stability study of the original system.

6.2 SUGGESTIONS FOR FUTURE RESEARCH:

On the basis of the investigations carried out in this thesis, some of the unsolved problems that need further attention are briefly reviewed now. These are:

1. The decomposition achieved in Chapter III is purely mathematical. Each of the subsystems so obtained represents the dynamics of a 2-machine group. The number of subsystems derived thus becomes large as the number of machines increase. Alternative methods of decomposition may be tried.

2. The stability region for the overall system is derived via a scalar Lyapunov function. An alternative method of obtaining this region employing the stability boundaries of the individual subsystems, as suggested by Weissenberger [77] may be persued.

3. The conventional model of the system with a constant voltage behind the transient reactance has been only used in Chapter III. This simplicity in the model has been retained in order to demonstrate the applicability of the method to the power system problem. Further work is needed to include refinements like the governor control, voltage regulators, saliency etc., The method of decomposition proposed in this chapter will be useful for this purpose.

4. The concept of a corner point in Chapter IV yields better stability domains than those obtained by other methods in the literature. It has been shown that this point can be identified with the machine going out of step with respect to all other machines. This, however, requires further theoretical investigation. A recent paper of Gupta and El-Abiad [82] discusses improvement in the stability region by considering 2-machines going out of step etc. Examination of this heuristic concepts in the light of the theoretical results of Chapter IV is a subject of further research. Also the concepts of open Lyapunov surfaces introduced by Willems may be tried to improve the stability boundary.

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APPENDIX A

SOME DEFINITIONS IN CONNECTION WITH THEOREM 3.1

In this appendix, some definitions of terms used in theorem 3.1 are given [73].

DEFINITION A.1: A real valued function $\phi(r)$ belongs to class κ if it is defined, continuous and strictly increasing on $0 \leq r \leq r_1$ and if it vanishes at $r = 0$.

DEFINITION A.2: A function $V(\underline{X})$ is called positive definite in a domain $\mathcal{B}$ if there exists a real valued function $\phi(r)$ belonging to class κ such that

$$\phi(\|\underline{X}\|) \leq V(\underline{X}) \; , \quad \underline{X} \in \mathcal{B}. \tag{A.1}$$

DEFINITION A.3: A function $V(\underline{X})$ is called decrescent if there exists a function $\phi(r)$ of class κ such that in the neighbourhood of the origin and for all $t \geq t_o$

$$V(\underline{X}) \leq \Phi(\|\underline{X}\|) \tag{A.2}$$

DEFINITION A.4: A function $V(\underline{X})$ is said to be negative definite if $-V(\underline{X})$ is positive definite.

DEFINITION A.5: An autonomous system

$$\dot{\underline{X}} = \underline{F}(\underline{X}) \; ; \quad \underline{F}(\underline{0}) = \underline{0} \tag{A.3}$$

is said to be 'stable' in the sense of Lyapunov if for each $\varepsilon > 0$, there exists an $\eta > 0$ such that

$$\| \underline{X}(t) \| < \varepsilon \tag{A.4}$$

is valid whenever

$$\| \underline{X}(t_0) \| < \eta,$$

where $\underline{X}(t)$ is a solution of (A.3).

DEFINITION A.6: The autonomous system (A.3) is asymptotically stable whenever the origin $\underline{X} = \underline{0}$ is stable and furthermore for some $\varepsilon > 0$, every solution
sufficiently close to Origin
$\underline{X}(t) \to 0$ as $t \to \infty$.

APPENDIX B

PROOF OF THEOREM 3.2 OF CHAPTER III

In this appendix, the proof of theorem 3.2 is given [48].

Let us introduce a decrescent positive definite function $V_s(\underline{V}_v)$:

$$V_s(\underline{V}_v) = \underline{b}^T \underline{V}_v \quad (B.1)$$

where $\underline{b}$ is a constant s-vector $(b_1, b_2, \ldots b_s)^T$ and is positive element by element which we denote by $\underline{b} > 0$. Consider $V_s(\underline{V}_v)$ as a candidate for the Lyapunov's function for the composite system $\mathcal{S}$. Taking the total time derivative along the solution $\underline{X}(t; t_o, \underline{X}_o)$ of $\mathcal{S}$ we get

$$\dot{V}_s(\underline{V}_v) = \underline{b}^T \dot{\underline{V}}_v . \quad (B.2)$$

Premultiplying the inequality (3.14) by $\underline{b}^T > 0$ we get

$$\dot{V}_s(\underline{V}_v) \leq \underline{b}^T R \underline{W} . \quad (B.3)$$

Since R has non-negative off diagonal elements and satisfies conditions (3.18) R is Hurwitz [74]. Therefore conditions (3.18) are necessary and sufficient for the existence of an s-vector $\underline{b} > 0$ for any s-vector $\underline{c} > 0$ such that

$$\underline{c}^T = -\underline{b}^T R \quad (B.4)$$

This implies

$$\dot{V}_s(\underline{V}_v) \leq -\underline{c}^T \underline{W} \tag{B.5}$$

Since $\underline{c} > 0$ and $\underline{W} > 0$, $\dot{V}_s < 0$. Also since $\underline{b} > 0$, $\dot{V}_s(\underline{V}_v) > 0$. Thus the global asymptotic stability of $\mathcal{S}$ is assured. This completes the proof of the theorem 3.2 of Chapter III.

APPENDIX C

RESULTS OF APPLICATION OF THE MOORE-ANDERSON'S THEOREM TO THE POWER SYSTEM PROBLEM

In this appendix the theorem due to Moore-Anderson [81] is stated without proof. Results of application of this theorem to the power system in a manner outlined in reference [10], to derive a Lyapunov function $V(\underline{X})$ of the quadratic in state variables plus integral of the nonlinearity type are included.

THE MOORE-ANDERSON THEOREM: If there exist diagonal matrices $\alpha = \text{diag}(\alpha_i)$, $i = 1,2,\ldots m$ and $Q = \text{diag}(q_i)$, $i = 1,2,\ldots m$ with $\alpha_i \geq 0$, $q_i \geq 0$ and $\alpha_i + q_i > 0$ and $-\alpha_i/q_i$ is not a pole of the ith row of $W(s)$ such that

$$Z(s) = \alpha\, G^{-1} + (\alpha + Q(s))\, W(s) \tag{C.1}$$

is positive real, then the system described by the set of equations

$$\begin{aligned} \dot{\underline{X}} &= A\, \underline{X} - B\, \underline{f}(\underline{\sigma}) \\ \underline{\sigma} &= C^T\, \underline{X} \end{aligned} \tag{C.2}$$

is stable.

In equation (4.29), the matrix $G = \text{diag}(g_i)$, $i = 1,2,\ldots m$ with

$$0 \leq \sigma_i \, f_i(\sigma_i) \leq g_i \, \sigma_i^2 \quad \text{for } g_i > 0, \; i = 1,2,\ldots m. \tag{C.3}$$

For the application of the theorem one needs the following definition of positive real functions:

DEFINITION C.1: An mxm matrix $Z(s)$ is positive real if and only if the following conditions are satisfied.

1. $Z(s)$ is real rational
2. $Z(s)$ has no poles in Re $s > 0$
3. Poles of $Z(s)$ on Re $s = 0$ are simple
4. For each pole on Re $s = 0$, the residue matrix $\hat{R}$ is hermitian and positive semidefinite,

and

5. The hermitian part $Z_H(j\omega)$ of $Z(s)$ is positive semidefinite, i.e.

$$Z_H(j\omega) + Z_H^T(-j\omega) \geq 0 \quad \text{for all real } \omega.$$

Proof of the above theorem is contained in reference [81].

Utilizing theorem 1 and the above definition on positive real functions systematic construction of Lyapunov function for the system (4.40) resulted [10] in a Lyapunov function of the form (4.41) where the matrix P given by

$$P = \left[\begin{array}{c|c} \psi^* & T \\ \hline T^T & \psi \end{array}\right] \tag{C.4}$$

and β_i are

(i) Uniform damping case:

$$P_{(2n-2)(2n-2)} = \left[\begin{array}{c|c} \psi^*_{(n-1)(n-1)} & T_{(n-1)(n-1)} \\ \hline T^T_{(n-1)(n-1)} & \psi_{(n-1)(n-1)} \end{array}\right] \tag{C.5}$$

where

$$T_{(n-1)(n-1)} = \frac{1}{\sum\limits_{i=1}^{n} M_i} \begin{bmatrix} M_2 \sum\limits_{\substack{j=1 \\ j\neq 2}}^{n} M_j & -M_2M_3 & \cdots & -M_2M_n \\ -M_3M_2 & M_3 \sum\limits_{\substack{j=1 \\ j\neq 3}}^{n} M_j & \cdots & -M_3M_n \\ \sigma. & \cdots & \cdots & \cdots \\ -M_nM_2 & -M_nM_3 & \cdots & M_n \sum\limits_{j=1}^{n-1} M_j \end{bmatrix}$$

(C.6)

$$\psi^*_{(n-1)(n-1)} = \frac{n}{\lambda} \psi_{(n-1)(n-1)} \tag{C,7}$$

and

$$\psi_{(n-1)(n-1)} = \lambda T_{(n-1)(n-1)} \tag{C.8}$$

and $\beta_i = \frac{2n}{\lambda}$ for all $i = 1,2,\ldots m$. (C.9)

(ii) Non-uniform damping case:

Here

$$P_{(2n-1)(2n-1)} = \left[\begin{array}{c|c} \psi^*_{nn} & T_{n(n-1)} \\ \hline T^T_{n(n-1)} & \psi_{(n-1)(n-1)} \end{array}\right] \tag{C.10}$$

where

$$\psi_{(n-1)(n-1)} = \frac{1}{\sum_{i=1}^{n} D_i} \begin{bmatrix} D_2 \sum_{\substack{j=1 \\ j\neq 2}}^{n} D_j & -D_2D_3 & \cdots & -D_2D_n \\ -D_2D_3 & D_3 \sum_{\substack{j=1 \\ j\neq i}}^{n} D_j & \cdots & -D_3D_n \\ \cdots & \cdots & \cdots & \cdots \\ -D_nD_2 & -D_nD_3 & \cdots & D_n \sum_{j=1}^{n-1} D_j \end{bmatrix} \tag{C.11}$$

$$T_{n(n-1)} = \lambda^{-1}_{nn} K_{n(n-1)} \psi_{(n-1)(n-1)} \tag{C.12}$$

and $$\psi^*_{nn} = \left(\sum_{i=1}^{n} \frac{1}{\lambda_i}\right) M . \tag{C.13}$$

The constant β_i is given by

$$\beta_i = 2\left(\sum_{i=1}^{n} \frac{1}{\lambda_i}\right) . \tag{C.14}$$

APPENDIX D

DATA FOR THE 5 MACHINE SYSTEM

The line constants and machine data are given in Tables D.1 and D.2. The load flow data prior to the fault on the system is given in Table D.3. All data are given in p.u.

Table D.1: Line Data

Line No.	Bus code		Impedance		Shunt susceptance
	from bus	to bus	R	X	B
1	1	2	0.0092	0.0582	0.0348
2	2	3	0.0000	0.0400	0.0000
3	3	4	0.0142	0.0970	0.3490
4	2	8	0.0098	0.0408	0.0090
5	2	7	0.0262	0.1094	0.0060
6	2	6	0.0204	0.1420	0.0270
7	2	5	0.0310	0.0882	0.0050
8	5	6	0.0200	0.1236	0.0080
9	6	7	0.0118	0.0643	0.0227
10	8	9	0.0080	0.0180	0.0900
11	8	10	0.0350	0.1340	0.0260
12	10	11	0.0350	0.1340	0.0265
13	9	11	0.0010	0.0040	0.0170
14	10	12	0.0196	0.0610	0.1260
15	1	12	0.0240	0.0862	0.0203

Table D.2: Machine Data

Machine No.	At bus	Inertia constant (H)	Equivalent impedance (p.u.) R	X
1	1	2.30	0.0420	0.1570
2	4	10.30	0.0000	0.1361
3	6	12.72	0.0000	0.0717
4	9	27.00	0.0000	0.0326
5	12	∞	0.0060	0.0280

Table D.3: Pre-fault Load Flow Conditions.

Bus No.	Bus voltage Magnitude	Angle (deg.)	Generation Real	Reactive	Load Real	Reactive
1	0.996	0.1	0.900	0.091	0.170	0.000
2	1.007	-2.0	0.000	0.000	1.400	0.200
3	0.912	2.8	0.000	0.000	0.000	0.000
4	1.000	12.8	1.760	0.442	0.000	0.000
5	1.003	-3.8	0.000	0.000	0.900	0.000
6	1.024	0.1	3.200	0.511	1.260	0.150
7	0.999	-3.7	0.000	0.000	1.350	0.200
8	0.992	-3.2	0.000	0.000	0.990	0.400
9	1.000	-2.9	3.500	1.462	2.500	0.900
10	0.963	-2.5	0.000	0.000	0.560	0.200
11	0.998	-2.0	0.000	0.000	0.560	0.200
12	0.956	0.0	0.443	-0.723	0.000	0.000

APPENDIX E

DATA FOR THE 44 BUS SYSTEM

The bus conditions in the pre-fault state are given in Table E.1. Line and shunt capacitor data are shown in Tables E.2 and E.3 respectively. The generator data is given in Table E.4. All data is in p.u. on a 200 MVA base. Damping constants of all machines are neglected.

Table E.1: Pre-fault Load Flow Conditions.

Bus No.	Bus voltage		Generation		Load	
	Magnitude	Phase angle (deg.)	Real	Reactive	Real	Reactive
1	1.0300	0.0000	4.1087	0.4539	0.0000	0.0000
3	1.0757	-4.2430	0.0000	0.0000	0.0000	0.0000
3	1.0250	-2.4140	2.3000	0.8542	0.0000	0.0000
4	1.0503	-6.6600	0.0000	0.0000	0.0000	0.0000
5	1.0497	-12.8020	0.0000	0.0000	0.8500	0.6000
6	1.0245	-16.8260	0.0000	0.0000	0.6000	0.3700
7	1.0000	-3.0020	1.5000	0.5200	0.0000	0.0000
8	1.0228	-8.0380	0.0000	0.0000	1.5400	0.7450
9	1.0250	-9.6280	0.0000	0.0000	0.2683	0.2010
10	1.0378	-14.3620	0.0000	0.0000	1.5000	1.0005
11	1.1892	-18.3320	0.0690	0.0000	0.0000	0.0000
12	1.1348	-21.9430	0.0000	0.0000	1.1750	0.8000
13	1.0240	- 1.4560	0.4950	0.1793	0.0000	0.0000
14	1.0405	-7.6740	0.0000	0.0000	0.0900	0.0435
15	1.0390	-33.3520	0.1000	0.0620	0.0000	0.0000
16	1.0800	-19.4570	1.0120	0.4096	0.0000	0.0000

(continued)

Table E.1 continued.

17	1.0529	-23.4400	0.0000	0.0000	1.2500	0.7500
18	1.0594	-22.5460	0.0000	0.0000	0.9000	0.6500
19	1.0297	-27.3030	0.0000	0.0000	0.2000	0.1500
20	1.0354	-27.1950	0.0000	0.0000	0.6255	0.4690
21	1.0223	-36.4070	0.0000	0.0000	0.2250	0.0750
22	1.0077	-26.9160	0.0000	0.0000	0.4500	0.3380
23	1.0832	-21.8800	0.1225	0.0000	0.0000	0.0000
24	1.0500	-12.5550	0.8350	0.1471	0.0000	0.0000
25	1.0646	-17.5740	0.0000	0.0000	0.0000	0.0000
26	1.0360	-20.3840	0.0000	0.0000	0.2315	0.1735
27	1.0098	-25.0910	0.0000	0.0000	0.7570	0.5650
28	0.9900	-20.1920	1.2400	0.8687	0.0000	0.0000
29	1.0154	-26.6640	0.0000	0.0000	0.9460	0.7090
30	0.9890	-32.3770	0.0000	0.0000	0.0000	0.0000
31	1.0630	-7.2760	0.5080	0.1149	0.0000	0.0000
32	0.9926	-20.4150	0.0000	0.0000	0.2750	0.2100
33	1.1043	-12.1480	0.0850	0.0000	0.0000	0.0000
34	1.0462	-19.0310	0.0000	0.0000	0.1000	0.0750
35	1.0739	-12.9000	0.0000	0.0000	0.2500	0.1500
36	1.0723	-16.5140	0.0000	0.0000	0.4500	0.2200
37	1.0450	- 7.9030	1.5000	0.5398	0.0000	0.0000
38	1.0673	-13.0120	0.0000	0.0000	0.0000	0.0000
39	0.9746	-24.4070	0.0000	0.0000	0.3375	0.2525
40	1.0455	-15.1690	0.0000	0.0000	0.0000	0.0000
41	1.0390	-10.3350	0.7350	0.3650	0.0000	0.0000
42	1.0680	-13.2620	0.0000	0.0000	0.0000	0.0000
43	1.0400	-6.9430	1.0800	0.1122	0.0000	0.0000
44	0.9918	-28.8730	0.0000	0.0000	2.1975	1.3490

Table E.2: Line Data

Sl. No.	Bus Code from bus	Bus Code to bus	Impedance Resis-	Impedance Reactance	$\frac{1}{2}$B	Off nominal turns ratio
1	8	7	0.0000	0.0570	0.0000	1.050
2	8	14	0.0220	0.0520	0.0135	1.000
3	8	9	0.0520	0.1270	0.0140	1.000
4	9	10	0.0660	0.1610	0.0180	1.000
5	14	9	0.0270	0.0700	0.0070	1.000
6	10	12	0.1570	0.3860	0.0630	1.000
7	11	12	0.0000	1.1730	0.0000	1.050
8	14	13	0.0000	0.2220	0.0000	1.050
9	20	17	0.0250	0.1250	0.1130	1.000
10	14	3	0.0000	0.0330	0.0000	1.050
11	14	4	0.0000	0.0530	0.0000	1.000
12	4	10	0.1600	0.1310	0.0710	0.900
13	2	4	0.0000	0.0620	0.0000	1.000
14	4	6	0.0580	0.2860	0.0645	1.000
15	2	1	0.0000	0.0190	0.0000	1.050
16	6	17	0.0340	0.1670	0.1500	1.000
17	5	12	0.0440	0.2670	0.1010	0.850
18	2	5	0.0050	0.0510	0.6706	1.000
19	17	16	0.0000	0.0780	0.0000	1.000
20	17	18	0.0130	0.0640	0.0580	1.000
21	18	5	0.0620	0.1232	0.2012	1.150
22	19	22	0.0840	0.1880	0.0210	1.000
23	23	22	0.0770	0.7610	0.0215	1.050
24	22	27	0.1090	0.1960	0.0220	1.000
25	30	27	0.0000	0.0800	0.0000	0.950
26	20	29	0.0210	0.1030	0.0920	1.000
27	29	28	0.0000	0.0830	0.0000	1.100
28	29	44	0.1700	0.8400	0.0760	1.000

(continued)

29	30	29	0.0370	0.1950	0.0390	1.000
30	44	39	0.0290	0.1520	0.0300	1.000
31	39	32	0.0160	0.0850	0.0170	1.000
32	32	34	0.0560	0.3040	0.0035	0.950
33	24	25	0.0000	0.1200	0.0000	0.975
34	25	26	0.0370	0.0900	0.0100	1.000
35	26	27	0.0830	0.1540	0.0170	1.000
36	26	34	0.1070	0.1970	0.2100	1.000
37	40	36	0.0000	0.0800	0.0000	0.950
38	34	36	0.0900	0.2310	0.0060	1.000
39	40	43	0.0230	0.1420	0.1040	1.050
40	35	31	0.0000	0.1880	0.0000	1.025
41	38	37	0.0000	0.0630	0.0000	1.050
42	38	32	0.0280	0.1440	0.0290	1.000
43	38	42	0.0080	0.0420	0.0080	1.000
44	38	40	0.0230	0.1190	0.0240	1.000
45	38	35	0.0000	0.1600	0.0000	1.000
46	36	35	0.0740	0.1880	0.0190	1.000
47	40	44	0.0074	0.1974	0.2415	1.000
48	25	22	0.3490	0.8900	0.0120	1.000
49	18	19	0.0800	0.4650	0.0420	0.975
50	15	21	0.2900	0.7830	0.0025	0.9500
51	17	21	0.3980	1.8960	0.0265	0.975
52	41	42	0.0000	0.0810	0.0000	0.950
53	42	40	0.0100	0.0480	0.0430	1.000
54	40	30	0.0230	0.2300	0.0695	1.000
55	34	33	0.1170	1.7150	0.0010	0.950
56	4	6	0.0580	0.2860	0.0645	1.000

Table E.3: Shunt Capacitor Data

Sl.No.	Bus No.	Admittance	
		G	B
1	5	0.0	0.3400
2	12	0.0	0.6100
3	17	0.0	0.3750
4	18	0.0	0.2900
5	19	0.0	0.1100
6	20	0.0	0.4200
7	22	0.0	0.2200
8	27	0.0	0.1900
9	44	0.0	1.0000

Table E.4: Machine Data

Sl.No.	At bus	Inertia constant 'H'	Transient reactance
1	1	43.7500	0.0309
2	3	15.4050	0.0689
3	7	6.6480	0.2010
4	11	0.3450	1.1733
5	13	1.9230	0.5278
6	15	0.5700	2.0553
7	16	8.2500	0.1965
8	23	0.6900	1.6232
9	24	2.2600	0.3520
10	28	4.2790	0.1185
11	31	2.9520	0.4045
12	33	0.4350	2.0000
13	37	6.1900	0.1500
14	41	4.8640	0.2555
15	43	4.3130	0.2319

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(2) 'Finite regions of attraction for multinonlinear systems and its application to the power system stability problem' (with M.A.Pai), Communicated to the IEEE Trans. on Aut.Control.

(3) Discussion on 'A simplified determination of transient stability regions for Lyapunov methods by F.S.Prabhakara and A.H.El-Abiad', (with M.A. Pai), IEEE Trans. Power Apparatus and Systems, Vol.PAS-94, No.2, March/April 1975.

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\+ Not pertaining to this thesis.